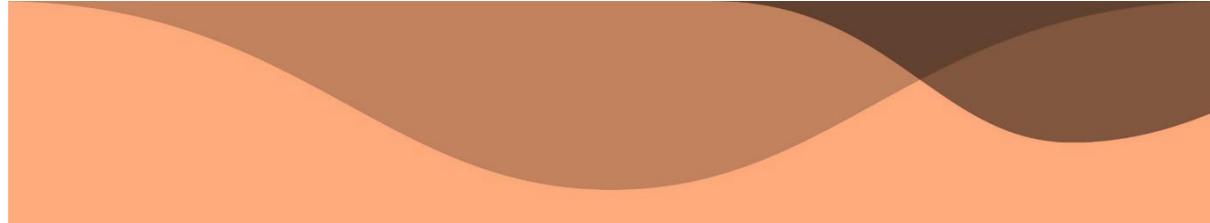


KS3 STAGE 9, ALGEBRA, MATHS

Rationale and Context of Unit:	Core curriculum content:	Tier 2 & Tier 3 vocabulary explicitly taught:
<p><b>Algebraic Proficiency: Tinkering</b></p> <ul style="list-style-type: none"> <li>Manipulate expressions by collecting like terms</li> <li>Know that <math>x \times x = x^2</math></li> <li>Calculate with negative numbers</li> <li>Know the grid method for multiplying two two-digit numbers</li> </ul> <p>Know the difference between an expression, an equation and a formula</p> <p><b>Pattern Spotting</b></p> <ul style="list-style-type: none"> <li>Generate a linear sequence from its nth term</li> <li>Substitute positive numbers into quadratic expressions</li> <li>Find the nth term for an increasing linear sequence</li> <li>Find the nth term for an decreasing linear sequence</li> </ul> <p><b>Solving Equations and Inequalities I</b></p> <ul style="list-style-type: none"> <li>Understand the meaning of the four inequality symbols</li> <li>Solve linear equations including those with unknowns on both sides</li> </ul>	<p><b>Algebraic Proficiency: Tinkering</b></p> <ul style="list-style-type: none"> <li><b>Identities</b></li> <li>You will discover the difference between equations and identities</li> <li>Understand the meaning of an identity</li> <li>Work out why two algebraic expressions are equivalent</li> <li>Know how to set up an mathematical argument</li> <li>Create a mathematical argument to show that two algebraic expressions are equivalent</li> <li><b>Manipulating Quadratics</b></li> <li>You will learn how to manipulate quadratics expressions</li> <li>Multiply two linear expressions of the form <math>(x + a)(x + b)</math></li> <li>Multiply two linear expressions of the form <math>(x \pm a)(x \pm b)</math></li> <li>Expand the expression <math>(x \pm a)^2</math></li> <li>Simplify an expression involving 'x<sup>2</sup>' by collecting like terms</li> <li>Identify when it is necessary to remove factors to factorise a quadratic expression</li> </ul>	<p>Inequality, Identity, Equivalent, <b>Equation</b>, Formula, Formulae, Expression, Expand, <b>Linear</b>, <b>Quadratic</b>. Term, Term-to-term rule, Position-to-term rule, nth term, Generate, First (second) difference, Fibonacci number, Fibonacci sequence. Unknown, Manipulate, <b>Solve</b>, Solution set, Integer. Congruent, congruence, Similar (shapes), similarity, <b>Hypotenuse</b>, Conjecture, Derive, <b>Prove</b>, <b>proof</b>, Counter-example. <b>Function</b>, non-linear, cubic, reciprocal, Parabola, Asymptote, Gradient, y-intercept, x-intercept, root, Rate of change, Sketch, plot, Kinematic, Speed, distance, time, Acceleration, Deceleration. <b>Simultaneous equation</b>, <b>Variable</b>, Manipulate, Eliminate, Interpret.</p> <p><b>Highlighted words MUST</b> be explicitly taught, defined and recorded in student books as they are first met. Other listed words may be introduced verbally or written in a similar format.</p>



<p><b>Proof</b></p> <ul style="list-style-type: none"> <li>• Know angle facts including angles at a point, on a line and in a triangle</li> <li>• Know angle facts involving parallel lines and vertically opposite angles</li> <li>• Know the properties of special quadrilaterals</li> </ul> <p>Know Pythagoras' theorem</p> <p><b>Algebraic Proficiency: Visualising</b></p> <ul style="list-style-type: none"> <li>• Plot straight-line graphs</li> <li>• Interpret gradients and intercepts of linear functions graphically and algebraically</li> <li>• Recognise, sketch and interpret graphs of linear functions</li> <li>• Recognise graphs of simple quadratic functions</li> </ul> <p>Plot and interpret graphs of kinematic problems involving distance and speed</p> <p><b>Solving Equations and Inequalities II</b></p> <ul style="list-style-type: none"> <li>• Solve linear equations</li> <li>• Substitute numbers into formulae</li> <li>• Plot graphs of functions of the form <math>y = mx + c</math>, <math>x \pm y = c</math> and <math>ax \pm by = c</math></li> </ul> <p>Manipulate expressions by multiplying by a single term</p>	<ul style="list-style-type: none"> <li>• Identify when it is necessary to find two linear expressions to factorise a quadratic expression</li> <li>• Factorise a quadratic expression of the form <math>x^2 + bx + c</math></li> <li>• <b><u>Formulae</u></b></li> <li>• You will use apply your algebra skills to create and use formulae in different situations</li> <li>• Construct algebraic statements</li> <li>• Identify variables in a situation</li> <li>• Distinguish between situations that can be modelled by an expression or a formula</li> <li>• Create an expression or a formula to describe a situation</li> </ul> <p><b>Pattern Spotting</b></p> <ul style="list-style-type: none"> <li>• <b><u>Nth term of a linear sequence</u></b></li> <li>• You will explore how do describe any term in a linear sequence</li> <li>• Generate a sequence from a term-to-term rule</li> <li>• Understand the meaning of a position-to-term rule</li> <li>• Use a position-to-term rule to generate a sequence</li> <li>• Find the position-to-term rule for a given sequence</li> </ul>	
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- Use algebra to describe the position-to-term rule of a linear sequence (the nth term)
- Use the nth term of a sequence to deduce if a given number is in a sequence
- Generate a sequence using a spreadsheet
- **Fibonacci Sequences**
- You will investigate Fibonacci numbers and Fibonacci type sequences
- Recognise Fibonacci numbers
- Recognise the Fibonacci sequence
- Generate Fibonacci type sequences
- Find the next three terms in any Fibonacci type sequence
- **Quadratic Sequences**
- You will explore quadratic sequences
- Substitute numbers into formulae including terms in  $x^2$
- Generate terms of a quadratic sequence from a written rule
- Generate terms of a quadratic sequence from its nth term
- Identify quadratic sequences
- Establish the first and second differences of a quadratic sequence
- Find the next three terms in any quadratic sequence

## **Solving Equations and Inequalities I**

### **Inequalities**

You will explore the meaning of an inequality and learn to solve inequalities

- Understand the meaning of the four inequality symbols
- Choose the correct inequality symbol for a particular situation
- Represent practical situations as inequalities
- Recognise a simple linear inequality
- Find the set of integers that are solutions to an inequality
- Use set notation to list a set of integers
- Use a formal method to solve an inequality
- Use a formal method to solve an inequality with unknowns on both sides
- Use a formal method to solve an inequality involving brackets
- Know how to deal with negative number terms in an inequality
- Know how to show a range of values that solve an inequality on a number line
- Know when to use an open circle at the end of a range of values shown on a number line
- Know when to use a filled circle at the end of a range of values shown on a number line

- Use a number line to find the set of values that are true for two inequalities

**Proof**

**Congruence**

You will explore the congruence of triangles and investigate geometrical situations

- Know the criteria for triangles to be congruent (SSS, SAS, ASA, RHS)
- Identify congruent triangles

**Proof**

You will learn how to form mathematical arguments and prove them mathematically

- Use known facts to form conjectures about lines and angles in geometrical situations
- Use known facts to derive further information in geometrical situations
- Test conjectures using known facts
- Know the structure of a simple mathematical proof
- Use known facts to create simple proofs
- Explain why the base angles in an isosceles triangle must be equal
- Explain the connections between Pythagorean triples

**Algebraic Proficiency: Visualising**  
**Linear Graphs**

You will investigate features of straight line graphs

- Use the form  $y = mx + c$  to identify parallel lines
- Rearrange an equation into the form  $y = mx + c$
- Find the equation of a line through one point with a given gradient
- Find the equation of a line through two given points
- Interpret the gradient of a straight line graph as a rate of change

**Curved Graphs**

You will explore graphs of quadratic functions and other nonlinear functions

- Explore graphs of other standard non-linear functions
- Create and use graphs of non-standard functions
- Plot graphs of quadratic (cubic, reciprocal) functions
- Recognise and interpret the graphs of quadratic (cubic, reciprocal) functions
- Sketch graphs of quadratic (cubic, reciprocal) functions
- Plot and interpret graphs of non-standard functions in real contexts

**Kinematics**

- You will use graphs to solve about objects in motion
- Plot and interpret graphs of non-standard functions in real contexts
- Find approximate solutions to kinematic problems involving distance, speed and acceleration

**Solving Equations and Inequalities II**  
**Simultaneous Equations**

You will learn how to solve simultaneous equations graphically and algebraically

- Understand that there are an infinite number of solutions to the equation  $ax + by = c$  ( $a \neq 0, b \neq 0$ )
- Understand the concept of simultaneous equations
- Find approximate solutions to simultaneous equations using a graph
- Understand the concept of solving simultaneous equations by elimination\*
- Target a variable to eliminate
- Decide if multiplication of one equation is required
- Decide whether addition or subtraction of equations is required
- Add or subtract pairs of equations to eliminate a variable
- Find the value of one variable in a pair of simple simultaneous equations

	<ul style="list-style-type: none"> <li>• Find the value of the second variable in a pair of simple simultaneous equations</li> <li>• Solve two linear simultaneous equations in two variables in very simple cases (no multiplication required)</li> <li>• Solve two linear simultaneous equations in two variables in simple cases (multiplication of one equation only required)</li> <li>• Derive and solve two simultaneous equations</li> <li>• Interpret the solution to a pair of simultaneous equations</li> </ul>	
<p><b>Challenge and Support:</b></p>	<p><b>World wide learning/ links to 21<sup>st</sup> century:</b></p>	<p><b>Cultural capital/ Industry/ Enrichment:</b></p>
<ul style="list-style-type: none"> <li>• The answer is <math>x^2 + 10x + c</math>. Show me a possible question. And another. And another ... (Factorising a quadratic expression of the form <math>x^2 + bx + c</math> can be introduced as a reasoning activity: once pupils are fluent at multiplying two linear expressions they can be asked 'if this is the answer, what is the question?')</li> <li>• Convince me that <math>(x + 3)(x + 4)</math> does not equal <math>x^2 + 7</math>.</li> <li>• What is wrong with this statement? How can you correct it? <math>(x + 3)(x + 4) \equiv x^2 + 12x + 7</math>.</li> </ul> <p>Jenny thinks that <math>(x - 2)^2 = x^2 - 4</math>. Do you agree with Jenny? Explain your answer.</p>	<ul style="list-style-type: none"> <li>• Situations that involve motion, including acceleration, stopping distances, velocity and displacement can be modelled using quadratic expressions and formulae. Police road accident investigators measure skid marks and apply an equation to work out the speed at which the accident occurred.</li> <li>• Finding a pattern and working out how the parts of a pattern fit together is important in scientific discovery. Scientists use sequences to model and solve real life problems, such as estimating how quickly a disease will spread.</li> <li>• Inequalities are one way of showing the ranges of values that have to be met and considered in running a successful business.</li> </ul>	<p style="text-align: center;">Search Algebra ages 11-14</p> <p>NRICH website – access current articles and enrichment activities.</p> <div style="display: flex; align-items: center; justify-content: center;">  <div style="margin-left: 20px;"> <ul style="list-style-type: none"> <li>• NRICH provides thousands of free online mathematics resources for ages 3 to 18 - completely free and available to all via their website (<a href="http://nrich.maths.org/">nrich.maths.org/</a>). These resources aim to:</li> </ul> </div> </div>

- A sequence has the first two terms 1, 2, ... Show me a way to continue this sequence. And another. And another ...
- A sequence has nth term  $3n^2 + 2n - 4$ . Jenny writes down the first three terms as 1, 12, 29. Kenny writes down the first three terms as 1, 36, 83. Who do agree with? Why? What mistake has been made? What is the same and what is different: 1, 1, 2, 3, 5, 8, ... and 4, 7, 11, 18, 29, ...
- Show me an inequality (with unknowns on both sides) with the solution  $x \geq 5$ . And another. And another ...
- Convince me that there are only 5 common integer solutions to the inequalities  $4x < 28$  and  $2x + 3 \geq 7$ . What is wrong with this statement? How can you correct it?  $1 - 5x \geq 8x - 15$  so  $1 \geq 3x - 15$ .
- Show me a pair of congruent triangles. And another. And another
- Show me a pair of similar triangles. And another. And another
- What is the same and what is different: Proof, Conjecture, Justification, Test?
- Convince me the base angles of an isosceles triangle are equal.
- Show me a Pythagorean Triple. And another. And another.

For example, a business might want wastage to be below a certain figure or profit to be greater than or equal to a given figure.

- Mathematical proof is proceeding in logical steps, establishing a series of mathematical statements by using facts that are already known to be true.
- Mathematical proof is proceeding in logical steps, establishing a series of mathematical statements by using facts that are already known to be true.
- Without algebra, humans would not have reached the moon and planes would not fly. Defining numbers with letters allows mathematicians to use formulae and solve very complicated equations that are needed for today's technology.

- Enrich and enhance the experience of the mathematics curriculum for all learners
- Develop mathematical thinking and problem-solving skills
- Offer challenging, inspiring and engaging activities
- Transition Day – Cluedo activity using Coordinates.
- Code breaking
- Functional Skills Projects

<p>Convince me a triangle with sides 3, 4, 5 is right-angled but a triangle with sides 4, 5, 6 is not right-angled.</p> <ul style="list-style-type: none"> <li>• Convince me the lines <math>y = 3 + 2x</math>, <math>y - 2x = 7</math>, <math>2x + 6 = y</math> and <math>8 + y - 2x = 0</math> are parallel to each other.</li> <li>• What is the same and what is different: <math>y = x</math>, <math>y = x^2</math>, <math>y = x^3</math> and <math>y=1/x</math> ?</li> <li>• Show me a sketch of a quadratic (cubic, reciprocal) graph. And another. And another ...</li> </ul> <p>Sketch a distance/time graph of your journey to school. What is the same and what is different with the graph of a classmate?</p> <ul style="list-style-type: none"> <li>• Show me a solution to the equation <math>5a + b = 32</math>. And another, and another ...</li> <li>• Show me a pair of simultaneous equations with the solution <math>x = 2</math> and <math>y = -5</math>. And another, and another ...</li> </ul> <p>Kenny and Jenny are solving the simultaneous equations <math>x + 4y = 7</math> and <math>x - 2y = 1</math>. Kenny thinks the equations should be added. Jenny thinks they should be subtracted. Who do you agree with? Explain why.</p>		
<p><b>Historical, Social, Moral, Spiritual, Cultural context:</b></p>	<p><b>Cross curricular links/ literacy/numeracy:</b></p>	<p><b>Common misconceptions:</b></p>
<ul style="list-style-type: none"> <li>• Algebra allows students to use spreadsheets, solve real world problems, use and understand modern</li> </ul>	<p>The lower case and upper case of a letter should not be used interchangeably when worked with algebra</p>	<ul style="list-style-type: none"> <li>• Once pupils know how to factorise a quadratic expression of the form <math>x^2 + bx + c</math> they might overcomplicate the</li> </ul>

technology and to work efficiently in the workplace. It is also fundamental to understanding patterns in the natural world.

Juxtaposition is used in place of 'x'.  $2a$  is used rather than  $a^2$ .

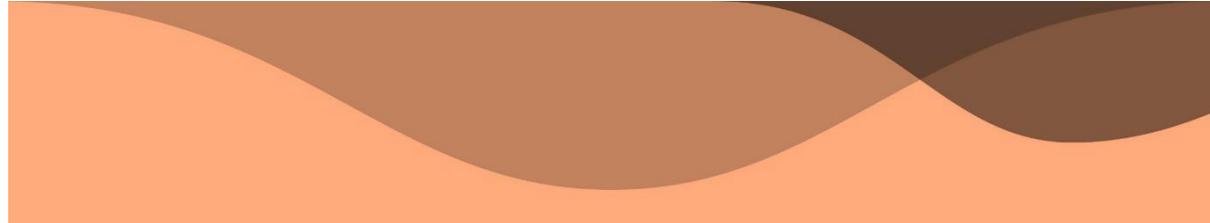
- Division is written as a fraction
- Algebra allows students to be able to communicate efficiently and to solve problems in Science (especially Physics)
- Correct use of specialised mathematical terms and phrases is crucial.

simpler case of factorising an expression such as  $x^2 + 2x$  ( $\equiv (x + 0)(x + 2)$ )

- Many pupils may think that  $(x + a)^2 \equiv x^2 + a^2$
- Some pupils may think that, for example,  $-2 \times -3 = -6$
- Some pupils may think that  $x^2 + 12 + 7x$  is not equivalent to  $x^2 + 7x + 12$ , and therefore think that they are wrong if the answer is given as  $x^2 + 7x + 12$
- Some students may think that it is possible to find an  $n$ th term for any sequence. A Fibonacci type sequence would require a recurrence relation instead.
- Some students may think that the word 'quadratic' involves fours.
- Some students may substitute into  $ax^2$  incorrectly, working out  $(ax)^2$  instead.
- Some pupils may think that it is possible to multiply or divide both sides of an inequality by a negative number with no impact on the inequality (e.g. if  $-2x > 12$  then  $x > -6$ )
- Some pupils may think that a negative  $x$  term can be eliminated by subtracting that term (e.g. if  $2 - 3x \geq 5x + 7$ , then  $2 \geq 2x + 7$ )

- Some pupils may know that a useful strategy is to multiply out any brackets, but apply incorrect thinking to this process (e.g. if  $2(3x - 3) < 4x + 5$ , then  $6x - 3 < 4x + 5$ )
- Some pupils think AAA is a valid criterion for congruent triangles.
- Some pupils try and prove a geometrical situation using facts that 'look OK', for example, '*angle ABC looks like a right angle*'.
- Some pupils do not appreciate that diagrams are often drawn to scale.
- Some pupils think that all triangles with sides that are consecutive numbers are right angled.
- Some pupils do not rearrange the equation of a straight line to find the gradient of a straight line. For example, they think that the line  $y - 2x = 6$  has a gradient of -2.
- Some pupils may think that gradient = (change in x) / (change in y) when trying to equation of a line through two given points.
- Some pupils may incorrectly square negative values of x when plotting graphs of quadratic functions.
- Some pupils think that the horizontal section of a distance time graph means

		<p>an object is travelling at constant speed.</p> <ul style="list-style-type: none"> <li>• Some pupils think that a section of a distance time graph with negative gradient means an object is travelling backwards or downhill.</li> <li>• Some pupils may think that addition of equations is required when both equations involve a subtraction</li> <li>• Some pupils may not multiply all coefficients, or the constant, when multiplying an equation</li> <li>• Some pupils may think that it is always right to eliminate the first variable</li> <li>• Some pupils may struggle to deal with negative numbers correctly when adding or subtracting the equations</li> </ul>
<p><b>Assessment timeline:</b></p>		
<ul style="list-style-type: none"> <li>• Topic test assessments (BAM tests) are conducted at the end of each topic. These are roughly after 2 weeks per topic, but this may vary.</li> <li>• Pre-checks are conducted at the start of the topic to test student prior knowledge. This informs lesson planning and delivery.</li> <li>• Tracking assessments are conducted once a term with end of year formal exams, for reporting and checking cumulative knowledge.</li> <li>• Testing data leads to discussions about setting, intervention groups and individual in-class intervention.</li> <li>• All students have access to a wide range of resources to develop their understanding.</li> </ul>		
<p><b>Home learning</b></p>		
<ul style="list-style-type: none"> <li>• Homework is set weekly for each group. This will often be via interactive websites with immediate feedback and support.</li> <li>• Teachers have the autonomy to use whichever resource they wish within the criteria set for the topic.</li> <li>• Students have access to lots of resources at home, including: Kerboodle, MyMaths, Mathswatch, PiXL Maths APP, PiXL Tmes Table App.</li> </ul>		



**Feedback**

Feedback is given after each topic test, tracking assessment and end of year exams. After tracking and end of year exams, this will include “Formative Marking” sheets which give feedback question by question to help support the students with priorities for further work.

**Length of unit (duration indicated in lessons)**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
61	62	63																											

**Unit:**