

YEAR 9, MASTERY, ALGEBRA, MATHS

Rationale and Context of Unit:	Core curriculum content:	Tier 2 & Tier 3 vocabulary explicitly taught:
<p>Prior key stage 3 content reviewed using pre-tests and whiteboard work at the start of each topic.</p> <p><b><u>ALGEBRAIC PROFICIENCY – TINKERING Indices</u></b>            You will learn about the notation of algebra</p> <ul style="list-style-type: none"> <li>• Know the multiplication (division, power, zero) law of indices</li> <li>• Understand that negative powers can arise</li> </ul> <p><b><u>Manipulating expressions</u></b>            You will learn that algebraic expressions can be written in different forms</p> <ul style="list-style-type: none"> <li>• Simplify an expression involving terms with combinations of variables (e.g. <math>3a^2b + 4ab^2 + 2a^2 - a^2b</math>)</li> <li>• Know how to write products algebraically</li> <li>• Use fractions when working in algebraic situations</li> <li>• Identify common factors (numerical and algebraic) of terms in an expression</li> <li>• Factorise an expression by taking out common factors</li> </ul>	<p><b><u>BASIC ALGEBRA REVIEW</u></b></p> <ul style="list-style-type: none"> <li>• Use and interpret algebraic notation</li> <li>• coefficients written as fractions rather than decimals</li> <li>• brackets</li> <li>• Use conventional notation for priority of operations, including brackets, powers, roots and reciprocals</li> <li>• understand and use the concepts and vocabulary of expressions, equations, formulae, <u>identities</u>, inequalities, terms and factors</li> <li>• Simplify and manipulate algebraic expressions by:               <ul style="list-style-type: none"> <li>• collecting like terms</li> <li>• multiplying a single term over a bracket</li> <li>• taking out common factors</li> </ul> </li> </ul> <p><b><u>COORDINATES AND LINEAR GRAPHS</u></b></p> <ul style="list-style-type: none"> <li>• Work with co-ordinates in all four quadrants</li> <li>• Solve geometrical problems on co-ordinate axes</li> <li>• Plot graphs of equations that correspond to straight line graphs in the co-ordinate plane</li> </ul>	<p>Algebra, brackets, <b>coefficient</b>, fraction, decimal, power, root, reciprocal, <b>equation</b>, formula, identity, inequality, term, factor, like term, single term, common factor, simplify, co-ordinate, quadrant, axes, y-axis, x-axis, graph, straight line, linear line, positive, negative, nth term, <b>sequence</b>, triangle number, square number, cube number, arithmetic sequence, quadratic sequence, Fibonacci sequence, geometric sequence, <b>integer</b>, <b>rational</b> number, linear sequence, plot, interpret, reciprocal, exponential, graph, <b>function</b>, distance, speed, acceleration, kinematics, gradient, rate of change, <b>substitution</b>, formula, expression, linear equation, <b>variable</b>, <b>constant</b>.</p> <p><b>Highlighted words <u>MUST</u></b> be explicitly taught, defined and recorded in student books as they are first met. Other listed words may be introduced verbally or written in a similar format.</p>

**Rearranging formulae**

Substitute positive and negative numbers into formulae

- Be aware of common scientific formulae
- Know the meaning of the 'subject' of a formula
- Change the subject of a formula when one step is required
- Change the subject of a formula when a two steps are required

**PATTERN SPOTTING**

**Nth term of a linear sequence**

You will explore how do describe any term in a linear sequence

- Generate a sequence from a term-to-term rule
- Understand the meaning of a position-to-term rule
- Use a position-to-term rule to generate a sequence
- Find the position-to-term rule for a given sequence
- Use algebra to describe the position-to-term rule of a linear sequence (the nth term)
- Use the nth term of a sequence to deduce if a given number is in a sequence

- Identify and interpret gradients and intercepts of linear functions graphically and algebraically.

**SEQUENCES**

- Generate terms of a sequence from either a term-to-term or a position-to-term rule
- Recognise and use:
  - sequences of triangular, square and cube numbers
  - simple arithmetic progression
  - Fibonacci type sequences
  - quadratic sequences
  - and simple geometric progressions ( $r^n$  where  $n$  is an integer and  $r$  is a rational number  $> 0$ )
- Other sequences
- Deduce expressions to calculate the  $n$ th term of a linear and quadratic sequence

**REAL LIFE GRAPHS**

- Plot and interpret graphs (including reciprocal graphs and exponential graphs) and graphs of non-standard functions in real context, to find approximate solutions to problems such as simple kinematics problems

- Generate a sequence using a spreadsheet

### **SOLVING EQUATIONS AND INEQUALITIES**

#### **Linear Equations**

You will learn how to solve linear equations with the unknown on one side and solve linear equations with the unknown on both sides

- Identify the correct order of undoing the operations in an equation
- Solve linear equations with the unknown on one side when the solution is a negative number
- Solve linear equations with the unknown on both sides when the solution is a whole number
- Solve linear equations with the unknown on both sides when the solution is a fraction
- Solve linear equations with the unknown on both sides when the solution is a negative number
- Solve linear equations with the unknown on both sides when the equation involves brackets
- Recognise that the point of intersection of two graphs corresponds to the solution of a connected equation  $n$
- Check the solution to an equation by substitution

involving distance, speed and acceleration.

- Interpret the gradient of a straight line as a rate of change.

#### **EQUATIONS**

- Substitute numerical values into formulae and expressions, including scientific formulae
- Solve linear equations in one unknown algebraically including those with the unknown on both sides of the equation

**ALGEBRAIC PROFICIENCY – VISUALISING**

**Linear graphs**

You will learn how to use a table of values to calculate points which satisfy a linear equation

- Know that graphs of functions of the form  $y = mx + c$ ,  $x \pm y = c$  and  $ax \pm by = c$  are linear
- Plot graphs of functions of the form  $y = mx + c$  ( $x \pm y = c$ ,  $ax \pm by = c$ )
- Understand the concept of the gradient of a straight line
- Find the gradient of a straight line on a unit grid
- Find the y-intercept of a straight line
- Sketch a linear graph

**Quadratic graphs**

You will learn to recognize and plot a simple quadratic graph

- Distinguish between a linear and quadratic graph
- Plot graphs of quadratic functions of the form  $y = x^2 \pm c$
- Sketch a simple quadratic graph

**Real life graphs**

- Plot and interpret graphs of piece-wise linear functions in real contexts
- Plot and interpret distance-time graphs (speed-time graphs)

<ul style="list-style-type: none"> <li>Find approximate solutions to kinematic problems involving distance and speed</li> </ul>		
<p><b>Challenge and Support:</b></p>	<p><b>World wide learning/ links to 21<sup>st</sup> century:</b></p>	<p><b>Cultural capital/ Industry/ Enrichment:</b></p>
<p><b>ALGEBRA</b></p> <p>Notation, vocabulary and manipulation</p> <p>1. use and interpret algebraic notation, including:</p> <ul style="list-style-type: none"> <li><math>ab</math> in place of <math>a \times b</math></li> <li><math>3y</math> in place of <math>y + y + y</math> and <math>3 \times y</math></li> <li><math>a^2</math> in place of <math>a \times a</math>, <math>a^3</math> in place of <math>a \times a \times a</math>, <math>a^2b</math> in place of <math>a \times a \times b</math></li> <li><math>\frac{a}{b}</math> in place of <math>a \div b</math></li> <li>coefficients written as fractions rather than as decimals</li> <li>brackets</li> </ul> <p>2. substitute numerical values into formulae and expressions, including scientific formulae</p> <p>3. understand and use the concepts and vocabulary of expressions, equations, formulae, identities inequalities, terms and factors</p> <p>4. simplify and manipulate algebraic expressions (including those involving surds <b>and algebraic fractions</b>) by:</p> <ul style="list-style-type: none"> <li>collecting like terms</li> <li>multiplying a single term over a bracket</li> </ul>	<ul style="list-style-type: none"> <li>You are unlikely to think about algebra when you watch cartoons or play video games, but animators use complex algebra to program the characters and make objects move.</li> <li>When designing a new building, an architect uses graphs to help identify and describe the structural properties the building needs to have.</li> <li>Finding a pattern and working out how the parts of the pattern fit together is important in scientific discovery. Scientists use sequences to model and solve real-life problems such as estimating how quickly diseases spread.</li> <li>All sorts of information can be obtained from graphs in real-life contexts. The shape of a graph, its gradient and the area underneath it can tell us about speed, time, acceleration, prices, earnings, break-even points or the value of one currency against another, among other things.</li> </ul>	<div data-bbox="1518 507 1771 687" data-label="Image"> </div> <p>Search Algebra for all ages</p> <p>NRICH website – access current articles and enrichment activities.</p> <ul style="list-style-type: none"> <li>NRICH provides thousands of free online mathematics resources for ages 3 to 18 - completely free and available to all via their website (<a href="http://nrich.maths.org/">nrich.maths.org/</a>). These resources aim to:             <ul style="list-style-type: none"> <li>Enrich and enhance the experience of the mathematics curriculum for all learners</li> <li>Develop mathematical thinking and problem-solving skills</li> <li>Offer challenging, inspiring and engaging activities</li> </ul> </li> <li>Problem solving opportunities – Applied Mathematics.</li> <li>Challenge problems.</li> </ul>

- taking out common factors
- expanding products of two **or more** binomials
- factorising quadratic expressions of the form  $x^2 + bx + c$ , including the difference of two squares; **factorising quadratic expressions of the form  $ax^2 + bx + c$**
- simplifying expressions involving sums, products and powers, including the laws of indices

5. understand and use standard mathematical formulae; rearrange formulae to change the subject

6. know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **and proofs**

7. where appropriate, interpret simple expressions as functions with inputs and outputs; **interpret the reverse process as the 'inverse function'; interpret the succession of two functions as a 'composite function'.**

Graphs

8. work with coordinates in all four quadrants

9. plot graphs of equations that correspond to straight-line graphs in the coordinate plane; use the form  $y = mx + c$  to identify parallel **and perpendicular lines**; find the equation of the

- Although the computer does the actual calculations, an accountant has to insert different equations to tell it what operations to perform and in which order to perform them.

- Extension work.
- Assessment sections in texts

line through two given points, or through one point with a given gradient

10. identify and interpret gradients and intercepts of linear functions graphically and algebraically

11. identify and interpret roots, intercepts, turning points of quadratic functions graphically; deduce roots algebraically **and**

**turning points by completing the square**

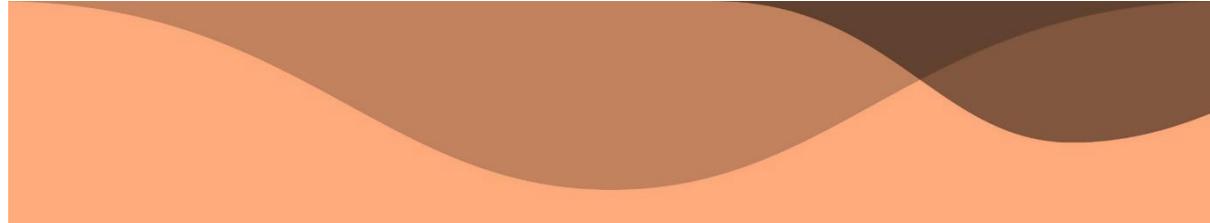
12. recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function  $y = \frac{1}{x}$

with  $x \neq 0$ , **exponential functions  $y = x^k$  for positive values of  $k$ , and the trigonometric functions (with arguments in degrees)  $y = \sin x$ ,  $y = \cos x$  and  $y = \tan x$  for angles of any size**

13. **sketch translations and reflections of a given function**

14. plot and interpret graphs (including reciprocal graphs **and exponential graphs**) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration

15. **calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as**



**distance-time graphs, velocity-time graphs and graphs in financial contexts**

**16. recognise and use the equation of a circle with centre at the origin; find the equation of a tangent to a circle at a given point.**

Solving equations and inequalities

**17. solve linear equations in one unknown algebraically (including those with the unknown on both sides of the equation); find approximate solutions using a graph**

**18. solve quadratic equations (including those that require rearrangement) algebraically by factorising, by completing the square and by using the quadratic formula; find approximate solutions using a graph**

**19. solve two simultaneous equations in two variables (linear/linear or linear/quadratic) algebraically; find approximate solutions using a graph**

**20. find approximate solutions to equations numerically using iteration**

**21. translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution.**

**22. solve linear inequalities in one or two variable(s), and quadratic inequalities in one variable; represent the solution set on a**

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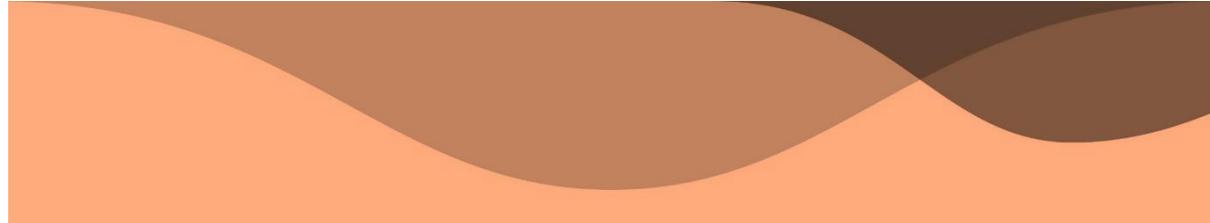
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<p>number line, <b>using set notation and on a graph</b> Sequences 23. generate terms of a sequence from either a term-to-term or a position-to-term rule 24. recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, Fibonacci type sequences, quadratic sequences, and simple geometric progressions (<math>r^n</math> where <math>n</math> is an integer, and <math>r</math> is a rational number <math>&gt; 0</math> <b>or a surd</b>) <b>and other sequences</b> 25. deduce expressions to calculate the <math>n</math>th term of linear <b>and quadratic</b> sequences.</p>		
<p><b>Historical, Social, Moral, Spiritual, Cultural context:</b></p>	<p><b>Cross curricular links/ literacy/numeracy:</b></p>	<p><b>Common misconceptions:</b></p>
<ul style="list-style-type: none"> <li>Algebra allows students to use spreadsheets, solve real world problems, use and understand modern technology and to work efficiently in the workplace. It is also fundamental to understanding patterns in the natural world.</li> </ul>	<p>The lower case and upper case of a letter should not be used interchangeably when worked with algebra Juxtaposition is used in place of 'x'. <math>2a</math> is used rather than <math>a^2</math>.</p> <ul style="list-style-type: none"> <li>Division is written as a fraction</li> <li>Algebra allows students to be able to communicate efficiently and to solve problems in Science (especially Physics)</li> <li>Correct use of specialised mathematical terms and phrases is crucial.</li> </ul>	<ul style="list-style-type: none"> <li>Some pupils may think that it is always true that <math>a=1</math>, <math>b=2</math>, <math>c=3</math>, etc.</li> <li>A common misconception is to believe that <math>a^2 = a \times 2 = a^2</math> or <math>2a</math> (which it can do on rare occasions but is not the case in general)</li> <li>When working with an expression such as <math>5a</math>, some pupils may think that if <math>a=2</math>, then <math>5a = 52</math>.</li> <li>Some pupils may think that <math>3(g+4) = 3g+4</math></li> <li>The convention of not writing a coefficient of 1 (i.e. '1x' is written as 'x')</li> </ul>

		<p>may cause some confusion. In particular some pupils may think</p> <ul style="list-style-type: none"> <li>• When describing a number sequence some students may not appreciate the fact that the starting number is required as well as a term-to-term rule</li> <li>• Some pupils may think that all sequences are ascending</li> <li>• Some pupils may think the <math>(2n)^{\text{th}}</math> term of a sequence is double the <math>n^{\text{th}}</math> term of a (linear) sequence</li> <li>• Some pupils may think that equations always need to be presented in the form <math>ax + b = c</math> rather than <math>c = ax + b</math>.</li> <li>• Some pupils may think that the solution to an equation is always positive and/or a whole number.</li> <li>• Some pupils may get the use the inverse operations in the wrong order, for example, to solve <math>2x + 18 = 38</math> the pupils divide by 2 first and then subtract 18.</li> </ul>
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**Assessment timeline:**

- Topic test assessments are conducted at the end of each topic. These are roughly after 2 weeks per topic, but this may vary.
- Pre-checks are conducted at the start of the topic to test student prior knowledge. This informs lesson planning and delivery.
- Tracking assessments are conducted once a term with end of year formal exams, for reporting and checking cumulative knowledge.
- Testing data leads to discussions about setting, intervention groups and individual in-class intervention.
- All students have access to a wide range of resources to develop their understanding.



## Home learning

- Homework is set weekly for each group. This will often be via interactive websites with immediate feedback and support.
- Teachers have the autonomy to use whichever resource they wish within the criteria set for the topic.
- Students have access to lots of resources at home, including: Kerboodle, MyMaths, Mathswatch, PiXL Maths APP, PiXL Times Table App.

## Feedback

- Feedback is given after each topic test, tracking assessment and end of year exams. After tracking and end of year exams, this will include “Formative Marking” sheets which give feedback question by question to help support the students with priorities for further work.

### Length of unit (duration indicated in lessons)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
<b>Unit:</b>																													