Further Maths Scheme and Specification

2a. OCR A Level in Further Mathematics B (MEI) (H645)

OCR's A Level in Further Mathematics B is a linear qualification in which all papers must be taken in the same examination series. To be awarded OCR's A Level in Further Mathematics B (MEI) learners must take one of three routes through the qualification, Route A, Route B or Route C.

Route A: Candidates must take the mandatory Core Pure and Mechanics Major papers and then one further optional minor paper. This paper must not be Mechanics Minor.
Route B: Candidates must take the mandatory Core Pure and Statistics Major papers and then one further optional minor paper. This paper must not be Statistics Minor.
Route C: Candidates must take the mandatory Core Pure paper and then three further minor optional papers.

Learners may **not** enter for Mechanics Major Y421 and Mechanics Minor Y431, Statistics Major Y422 and Statistics Minor Y432 or Mechanics Major Y421 and Statistics Major Y422.

Learners may take more than the required number of minor papers to increase the breadth of their course. For details of how their grade will be awarded, see Section 3g.

Content Overview

The qualification comprises one mandatory Core Pure paper and then a combination of optional papers:

- Core Pure content[1]
- Major options
  - Mechanics Major (Y421)[1]
  - Statistics Major (Y422)[1]
- Minor options
  - Mechanics Minor (Y431)[2]
  - Statistics Minor (Y432)[2]
  - Modelling with Algorithms (Y433)[2]
  - Numerical Methods (Y434)[2]
  - Extra Pure (Y435)
  - Further Pure with Technology (Y436)

The Overarching Themes must be applied along with associated mathematical thinking and understanding, across the whole of the subject content. See Section 2b.

[1]One third of the Core Pure content, and one half of the content of each major option can be co-taught with AS Further Mathematics. This material is labelled (a) throughout Sections 2c to 2e.

[2]These minor options can be co-taught with AS Further Mathematics.

Assessment Overview

Mandatory paper: Core Pure (Y420) 144 raw marks (180 scaled) 2 hour 40 mins Written paper	**50%** of total A level
Major Option 120 raw marks (120 scaled) 2 hour 15 mins Written paper	**33⅓%** of total A level
Minor Option 60 raw marks (60 scaled) 1 hour 15 mins Written paper (1 hour 45 mins Written paper for Y436)	**16⅔%** of total A level

Year 12 AS-level Further Maths Scheme of Work

Program of Study for groups taught by 2 teachers

Teacher 1 - Pure and Mechanics	Teacher 2 - Pure and Statistics
Ch1 - Matrices and Transformations Ch6 – Matrices and their inverses	Ch2 - Complex Numbers Ch3 - Roots of polynomials Ch4 - Sequences and Series
Ch7 – Vectors and 3D space An introduction to radians The identities sin(θ±ϕ) and cos(θ±ϕ) Revision and Assessment Practice Questions – Further Mathematics 1	Ch5 – Complex numbers and geometry Revision and Assessment Practice Questions – Further Mathematics 2
Ch1 – Kinematics Ch2 – Forces and motion Ch3 - A model for friction Ch4 Moments of forces	Ch1 - Statistical problem solving Ch2 - Discrete random variables Ch3 - Discrete probability distributions
Ch5 Work, energy and power Ch6 Impulse and momentum Ch7 Centre of mass 1 Ch8 Dimensional analysis	Ch 4 - Bivariate data (correlation coefficients) Ch 5 Bivariate data (regression lines) Ch 6 Chi-squared tests
Revision for Exam (1 Week) **End of Year 12 Assessment (1 week)**	Revision for Exams (1 Week) **End of Year 12 Assessment (1 week)**

Autumn - Half Term 1

Teacher 1

Ch1 - Matrices and Transformations (Integral link)

- Matrices (p2)
- Multiplication of matrices (p6)
- Transformations (p13)
- Successive transformations (p27)
- Invariance (p33)

LEARNING OUTCOMES

When you have completed this chapter you should be able to:

- understand what is meant by the terms order of a matrix, square matrix, identity matrix, zero matrix and equal matrices
- add and subtract matrices of the same order
- multiply a matrix by a scalar
- know when two matrices are conformable for multiplication, and be able to multiply conformable matrices
- use a calculator to carry out matrix calculations
- know that matrix multiplication is associative but not commutative
- find the matrix associated with a linear transformation in two dimensions:
  - reflections in the coordinate axes and the lines $y = \pm x$
  - rotations about the origin
  - enlargements centre the origin
  - stretches parallel to the coordinate axes
  - shears with the coordinate axes as fixed lines
- find the matrix associated with a linear transformation in three dimensions:
  - reflection in $x = 0$, $y = 0$ or $z = 0$
  - rotations through multiples of 90° about the x, y or z axes
- understand successive transformations in two dimensions and the connection with matrix multiplication
- find the invariant points for a linear transformation
- find the invariant lines for a linear transformation.

KEY POINTS

1. A matrix is a rectangular array of numbers or letters.
2. The shape of a matrix is described by its order. A matrix with r rows and c columns has order $r \times c$.
3. A matrix with the same number of rows and columns is called a square matrix.
4. The matrix $\mathbf{O} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ is known as the 2 × 2 zero matrix. Zero matrices can be of any order.
5. A matrix of the form $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is known as an identity matrix. All identity matrices are square, with 1s on the leading diagonal and zeros elsewhere.
6. Matrices can be added or subtracted if they have the same order.
7. Two matrices $\mathbf{A}$ and $\mathbf{B}$ can be multiplied to give matrix $\mathbf{AB}$ if their orders are of the form $p \times q$ and $q \times r$ respectively. The resulting matrix will have the order $p \times r$.

Ch6 – Matrices and their inverses (Integral link)

- The determinant of a matrix (p125)

Teacher 2

Ch2 – Complex numbers (Integral link)

- Introduction to complex numbers (p39)
- Extending the number system (p40)
- Division of complex numbers (p44)
- Representing complex numbers geometrically (p47)

LEARNING OUTCOMES

When you have completed this chapter you should be able to:

- understand how complex numbers extend the number system
- solve quadratic equations with complex roots
- know what is meant by the terms real part, imaginary part and complex conjugate
- add, subtract, multiply and divide complex numbers
- solve problems involving complex numbers by equating real and imaginary parts
- represent a complex number on an Argand diagram
- represent addition and subtraction of two complex numbers on an Argand diagram.

KEY POINTS

1. Complex numbers are of the form $z = x + yi$ with $i^2 = -1$.
   x is called the real part, $\mathrm{Re}(z)$, and y is called the imaginary part, $\mathrm{Im}(z)$.
2. The conjugate of $z = x + yi$ is $z^* = x - yi$.
3. To add or subtract complex numbers, add or subtract the real and imaginary parts separately.
   $(x_1 + y_1 i) + (x_2 + y_2 i) = (x_1 + x_2) + (y_1 + y_2)i$
4. To multiply complex numbers, expand the brackets then simplify using the fact that $i^2 = -1$
5. To divide complex numbers, write as a fraction, then multiply top and bottom by the conjugate of the bottom and simplify the answer.
6. Two complex numbers $z_1 = x_1 + y_1 i$ and $z_2 = x_2 + y_2 i$ are equal only if $x_1 = x_2$ and $y_1 = y_2$.
7. The complex number $z = x + yi$ can be represented geometrically as the point (x, y).
   This is known as an Argand diagram.

Ch3 – Roots of polynomials (Integral link)

- Polynomials (p53)
- Cubic equations (p58)
- Quartic equations (p62)
- Solving polynomial equations with complex roots (p65)

LEARNING OUTCOMES

When you have completed this chapter you should be able to:

- know the relationships between the roots and coefficients of quadratic, cubic and quartic equations
- form new equations whose roots are related to the roots of a given equation by a linear transformation
- understand that complex roots of polynomial equations with real coefficients occur in conjugate pairs
- solve cubic and quartic equations with complex roots.

KEY POINTS

1. If α and β are the roots of the quadratic equation $az^2 + bz + c = 0$, then
   $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$.
2. If α, β and γ are the roots of the cubic equation $az^3 + bz^2 + cz + d = 0$, then

- The inverse of a matrix (p131)
- Using matrices to solve simultaneous equations (p137)

LEARNING OUTCOMES

When you have completed this chapter you should be able to:

- find the determinant of a 2 × 2 matrix
- know what is meant by a singular matrix
- understand that the determinant of a 2 × 2 matrix represents the area scale factor of the corresponding transformation, and understand the significance of the sign of the determinant
- find the inverse of a non-singular 2 × 2 matrix
- use a calculator to find the determinant and inverse of a 3 × 3 matrix
- know that the determinant of a 3 × 3 matrix represents the volume scale factor of the corresponding transformation
- understand the significance of a zero determinant in terms of transformations
- use the product rule for inverse matrices
- use matrices to solve a pair of linear simultaneous equations in two unknowns
- use matrices to solve three linear simultaneous equations in three unknowns.

KEY POINTS

1. If $\mathbf{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then the determinant of $\mathbf{M}$, written $\det \mathbf{M}$ or $|\mathbf{M}|$ is given by $\det \mathbf{M} = ad - bc$
2. The determinant of a 2 × 2 matrix represents the area scale factor of the transformation.
   The determinant of a 3 × 3 matrix represents the volume scale factor of the transformation.
3. If $\mathbf{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then $\mathbf{M}^{-1} = \dfrac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$
4. $(\mathbf{MN})^{-1} = \mathbf{N}^{-1}\mathbf{M}^{-1}$
5. A matrix is singular if the determinant is zero. If the determinant is non-zero the matrix is said to be non-singular.
6. If the determinant of a matrix is zero, all points are mapped to either a straight line (in two dimensions) or to a plane (three dimensions).
7. If $\mathbf{A}$ is a non-singular matrix, $\mathbf{AA}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$.
8. When solving two simultaneous equations in two unknowns, the equations can be written as a matrix equation $\mathbf{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$.

   When solving three simultaneous equations in three unknowns, the equations can be written as a matrix equation $\mathbf{M}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$.

   In both cases, if $\det \mathbf{M} \neq 0$ there is a unique solution to the equations which can be found by pre-multiplying both sides of the equation by the inverse matrix $\mathbf{M}^{-1}$.
   If $\det \mathbf{M} = 0$ there is no unique solution to the equations. In this case there is either no solution or an infinite number of solutions.

Ch4 – Sequences and series (Integral link)

- Sequences and series (p72)
- Using standard results (p77)
- The method of differences (p80)
- Proof by induction (p85)
- Other proofs by induction (P90)

LEARNING OUTCOMES

When you have completed this chapter you should be able to:

- know what is meant by a sequence and a series
- find the sum of a series using standard formulae for $\sum r$, $\sum r^2$ and $\sum r^3$
- find the sum of a series using the method of differences
- use proof by induction to prove given results for the sum of a series
- use proof by induction to prove given results for the *n*th term of a sequence
- use proof by induction to prove given results for the *n*th power of a matrix.

KEY POINTS

1. The terms of a sequence are often written as $a_1, a_2, a_3, \ldots$ or $u_1, u_2, u_3, \ldots$
   The general term of a sequence may be written as a_r or u_r (sometimes the letters k or i are used instead of r). The last term is usually written as a_n or u_n.
2. A series is the sum of the terms of a sequence. The sum S_n of the first n terms of a sequence can be written using the symbol Σ (the Greek capital S, sigma).

   $$S_n = a_1 + a_2 + a_3 + \ldots + a_n = \sum_{r=1}^{n} a_r$$

   The numbers above and below the Σ are the limits of the sum. They show that the sum includes all the terms a_r from a_1 to a_n.
3. Some series can be expressed as combinations of these standard results:

   $$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1) \qquad \sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1) \qquad \sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

4. Some series can be summed by using the method of differences. If the terms of the series can be written as the difference of terms of another series, then many terms may cancel out. This is called a telescoping sum.
5. To prove by induction that a statement involving an integer n is true for all $n \geq n_0$, you need to:
   - prove that the result is true for an initial value of n_0, typically $n = 1$
   - find the target expression:
     use the result for $n = k$ to find the equivalent result for $n = k + 1$.
   - prove that:
     if it is true for $n = k$, *then* it is true for $n = k + 1$.
   - argue that since it is true for $n = 1$, it is also true for $n = 1 + 1 = 2$, and so for $n = 2 + 1 = 3$ and for all subsequent values of n.
   - conclude the argument with a precise statement about what has been proved.

Autumn - Half Term 2

Teacher 1	Teacher 2

Teacher 1

Ch7 – Vectors and 3D space

- Finding the angle between vectors (p143)
- The equation of a plane (p150)
- Intersection of planes (p157)

LEARNING OUTCOMES

When you have completed this chapter you should be able to:

- find the scalar product of two vectors
- use the scalar product to find the angle between two vectors
- know that two vectors are perpendicular if and only if their scalar product is zero
- identify a vector normal to a plane, given the equation of the plane
- find the equation of a plane in vector or Cartesian form
- find the angle between two planes
- know the different ways in which three distinct planes can be arranged in 3-D space
- understand how solving three linear simultaneous equations in three unknowns relates to finding the point of intersection of three planes in three dimensions.

KEY POINTS

1 In two dimensions, the scalar product

$$\mathbf{a}.\mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}.\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = a_1b_1 + a_2b_2 = |\mathbf{a}||\mathbf{b}|\cos\theta.$$

2 In three dimensions, $\mathbf{a}.\mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}.\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{a}||\mathbf{b}|\cos\theta.$

3 The angle θ between two vectors **a** and **b** is given by

$$\cos\theta = \frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

where $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2$ (in two dimensions)

$\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ (in three dimensions).

4 The cartesian equation of the plane perpendicular to the vector $\mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}$ and passing through a point with position vector **a** is given by $n_1x + n_2y + n_3z + d = 0$ where $d = -\mathbf{a}.\mathbf{n}$.

5 The vector equation of the plane through the point with position vector **a**, and perpendicular to the vector $\mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}$ is given by $(\mathbf{r} - \mathbf{a}).\mathbf{n} = 0$ or $\mathbf{r}.\mathbf{n} = \mathbf{a}.\mathbf{n}$.

6 The angle between two planes π_1 and π_2 is the same as the angle between their normals, $\mathbf{n}_1$ and $\mathbf{n}_2$. This angle can be found using the scalar product.

7 Three distinct planes in three dimensions will be arranged in one of five ways:

- meet in a unique point of intersection
- three parallel planes
- two parallel planes that are cut by the third to form two parallel lines
- a sheaf of planes that intersect in a common line
- a prism of planes in which each pair of planes meets in a straight line but there are no common points of intersection between the three planes.

8 Three distinct planes

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

can be expressed in the form

$$\mathbf{M}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$$

where $\mathbf{M} = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$.

If **M** is non-singular, the unique point of intersection is given by $\mathbf{M}^{-1}\begin{pmatrix} d_1 \\ d_2 \\ \ldots \end{pmatrix}$

Teacher 2

An introduction to radians (p169)

Ch5 – Complex numbers and geometry (Integral link)

- The modulus and argument (p98)
- Multiplying and dividing complex numbers in modulus-argument form (p106)
- Loci in the Argand diagram (p110)

LEARNING OUTCOMES

When you have completed this chapter you should be able to:

- find the modulus of a complex number
- find the principal argument of a complex number using radians
- express a complex number in modulus-argument form
- multiply and divide complex numbers in modulus-argument form
- represent multiplication and division of two complex numbers on an Argand diagram
- represent and interpret sets of complex numbers as loci on an Argand diagram:
  - circles of the form $|z - a| = r$
  - half-lines of the form $\arg(z - a) = \theta$
  - lines of the form $|z - a| = |z - b|$
- represent and interpret regions defined by inequalities based on the above.

KEY POINTS

1 The modulus of $z = x + yi$ is $|z| = \sqrt{x^2 + y^2}$. This is the distance of the point z from the origin on the Argand diagram.

2 The argument of z is the angle θ, measured in radians, between the line connecting the origin and the point z and the positive real axis.

3 The principal argument of z, $\arg z$, is the angle θ, measured in radians, for which $-\pi < \theta \leq \pi$, between the line connecting the origin and the point z and the positive real axis.

4 For a complex number z, $zz^* = |z|^2$.

5 The modulus-argument form of z is $z = r(\cos\theta + i\sin\theta)$, where $r = |z|$ and $\theta = \arg z$. This is often written as (r, θ)

6 For two complex numbers z_1 and z_2:

$$|z_1z_2| = |z_1||z_2| \qquad \arg(z_1z_2) = \arg z_1 + \arg z_2$$

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} \qquad \arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$$

7 The distance between the points z_1 and z_2 in an Argand diagram is $|z_1 - z_2|$.

8 $|z - a| = r$ represents a circle, centre a and radius r.
$|z - a| < r$ represents the interior of the circle, and $|z - a| > r$ represents the exterior of the circle.

9 $\arg(z - a) = \theta$ represents a half line starting at $z = a$ at an angle of θ from the positive real direction.

10 $|z - a| = |z - b|$ represents the perpendicular bisector of the points a and b.

The identities sin(θ±ϕ) and cos(θ±ϕ) (p172) **Revision and Assessment** Practice questions: Further Mathematics 1 **Past Exam Papers** https://www.ocr.org.uk/qualifications/as-and-a-level/further-mathematics-b-mei-h635-h645-from-2017/assessment/#as-level	**Revision and Assessment** Practice questions: Further Mathematics 2 **Past Exam Papers** https://www.ocr.org.uk/qualifications/as-and-a-level/further-mathematics-b-mei-h635-h645-from-2017/assessment/#as-level

Spring - Half Term 3

Teacher 1 - Mechanics

Ch1 – Kinematics (Integral link)

- The language of motion in one dimension (p2)
- The constant acceleration formulae (p11)
- Variable acceleration (p17)

KEY POINTS

1

Vectors (with magnitude and direction)	Scalars (magnitude only)
Displacement	Distance
Position: displacement from a fixed origin	
Velocity: rate of change of position	Speed: magnitude of velocity
Acceleration: rate of change of velocity	Magnitude of acceleration
	Time

2 Graphs

The gradient of a position–time graph is the velocity.
The gradient of a velocity–time graph is the acceleration.
The area under a velocity–time graph is the displacement.
The gradient of a distance–time graph is the speed.
The gradient of a speed–time graph is the magnitude of the acceleration.
The area under a speed–time graph is the distance.

3 Averages

$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

$$\text{Average velocity} = \frac{\text{displacement}}{\text{time taken}}$$

$$\text{Average acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

4 Constant acceleration formulae

$$v = u + at$$

$$s = \frac{1}{2}(u + v) \times t$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as.$$

5 Relationships between variables describing motion

Position	Velocity	Acceleration
$\longrightarrow$	differentiate	$\longrightarrow$
s	$v = \frac{ds}{dt}$	$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

Acceleration	Velocity	Position
$\longrightarrow$	integrate	$\longrightarrow$
a	$v = \int a \, dt$	$s = \int v \, dt$

LEARNING OUTCOMES

When you have completed this chapter, you should know

- the difference between position, displacement and distance travelled
- the difference between speed and velocity and between acceleration and the magnitude of acceleration
- how to draw and interpret position–time, distance–time, velocity–time, speed–time and acceleration–time graphs and how to use these to solve problems connected with motion in a straight line
- how to find average speed and average velocity
- how and when to use the constant acceleration formulae to solve problems involving linear motion
- how to deal with problems involving motion under gravity
- how to use calculus to derive expressions for position, velocity and acceleration as functions of time, given suitable information
- how to solve problems involving linear motion with variable acceleration.

Teacher 2 - Statistics

Ch1 - Statistical problem solving (Integral link)

- The problem solving cycle (p2)

KEY POINTS

1 The problem solving cycle has four stages:
- problem specification and analysis
- information collection
- processing and representation
- interpretation.

2 Information collection often involves taking a sample.

3 There are several reasons why you might wish to take a sample:
- to help you understand a situation better
- as part of a pilot study to inform the design of a larger investigation
- to estimate the values of the parameters of the parent population
- to avoid the work involved in cleaning and formatting all the data in a large set
- to conduct a hypothesis test.

4 Sampling procedures include:
- simple random sampling
- stratified sampling
- cluster sampling
- systematic sampling
- quota sampling
- opportunity sampling
- self-selected sampling.

5 For processing and representation, it is important to know the type of data you are working with i.e.:
- categorical data
- ranked data
- discrete numerical data
- continuous numerical data
- bivariate data.

6 Display techniques and summary measures must be appropriate for the type of data.

7 Notation for mean and standard deviation.

Mean $\bar{x} = \Sigma \frac{x_i}{n}$

Sum of squares $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - n\bar{x}^2$

Variance $s^2 = \frac{S_{xx}}{n-1}$

Standard deviation $s = \sqrt{\text{variance}} = \sqrt{\frac{S_{xx}}{n-1}}$.

LEARNING OUTCOMES

When you have completed this chapter you should be able to:

- use statistics within a problem solving cycle
- explain why sampling may be necessary in order to obtain information about a population, and give desirable features of a sample, including the size of the sample
- know a variety of sampling methods, the situations in which they might be used and any problems associated with them
- explain the advantage of using a random sample when inferring properties of a population
- display sample data appropriately
- calculate and interpret summary measures for sample data.

Ch2 – Forces and motion (Integral link)

- Forces and Newton's laws of motion (p26)
- Working in vectors (p34)
- Forces in equilibrium (p42) (Integral link)
- Finding resultant forces (p50)

KEY POINTS

1

Vectors (with magnitude and direction)	Scalars (magnitude only)
Displacement	Distance
Position: displacement from a fixed origin	
Velocity: rate of change of position	Speed: magnitude of velocity
Acceleration: rate of change of velocity	Magnitude of acceleration
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2 Graphs

The gradient of a position-time graph is the velocity.
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The gradient of a speed-time graph is the magnitude of the acceleration.
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$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

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5 Relationships between variables describing motion

Position	Velocity	Acceleration
$\longrightarrow$	differentiate	$\longrightarrow$
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Acceleration	Velocity	Position
$\longrightarrow$	integrate	$\longrightarrow$
a	$v = \int a\,dt$	$s = \int v\,dt$

LEARNING OUTCOMES

When you have completed this chapter, you should know

- the difference between position, displacement and distance travelled
- the difference between speed and velocity and between acceleration and the magnitude of acceleration
- how to draw and interpret position-time, distance-time, velocity-time, speed-time and acceleration-time graphs and how to use these to solve problems connected with motion in a straight line
- how to find average speed and average velocity
- how and when to use the constant acceleration formulae to solve problems involving linear motion
- how to deal with problems involving motion under gravity
- how to use calculus to derive expressions for position, velocity and acceleration as functions of time, given suitable information
- how to solve problems involving linear motion with variable acceleration.

Ch2 - Discrete random variables (Integral link)

- Notation and conditions for a discrete random variable (p22)
- Expectation and variance (p26)

KEY POINTS

1 For a *discrete random variable*, X, which can take only the values $r_1, r_2, \ldots, r_n$, with probabilities $p_1, p_2, \ldots, p_n$, respectively:

- $p_1 + p_2 + \cdots + p_n = \sum_{k=1}^{n} p_k = \sum_{1}^{n} P(X = r_k) = 1 \quad p_k \geq 0$

2 A discrete probability distribution is best illustrated by a *vertical line chart*.

- The *expectation* $= E(X) = \mu = \Sigma r P(x = r)$.
- The *variance* $= \text{Var}(X) = \sigma^2$

$$= E(X-\mu)^2 = \Sigma(r-\mu)^2 P(X=r)$$

$$\text{or} \quad E(X^2) - [E(X)]^2 = \Sigma r^2 P(X=r) - [\Sigma r P(X=r)]^2.$$

3 For a discrete random variable, X, which can assume only the values $x_1, x_2, \ldots, x_n$ with probabilities $p_1, p_2, \ldots, p_n$ respectively:

- $\Sigma p_i = 1 \qquad p_i \geq 0$
- $E(X) = \Sigma x_i p_i = \Sigma x_i \times P(X = x_i)$
- $\text{Var}(X) = E(X^2) - [E(X)]^2$.

4 For any random variables X and Y and constants a, b and c:

- $E(c) = c$
- $E(aX) = aE(X)$
- $E(aX + c) = aE(X) + c$
- $E(X \pm Y) = E(X) \pm E(Y)$
- $E(aX + bY) = aE(X) + bE(Y)$.

5 For two random variables X and Y, and constants a, b and c

- $\text{Var}(c) = 0$
- $\text{Var}(aX) = a^2\text{Var}(X)$
- $\text{Var}(aX + c) = a^2\text{Var}(X)$.

and, if X and Y are independent,

- $\text{Var}(X \pm Y) = \text{Var}(X) + \text{Var}(Y)$
- $\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y)$.

LEARNING OUTCOMES

When you have completed this chapter you should be able to:

- use probability functions, given algebraically or in tables
- calculate the numerical probabilities for a simple distribution
- draw and interpret graphs representing probability distributions
- calculate the expectation (mean), $E(X)$, and understand its meaning
- calculate the variance, $\text{Var}(X)$, and understand its meaning
- use the result $E(aX + b) = aE(X) + b$ and understand its meaning
- use the result $\text{Var}(aX + b) = a^2\text{Var}(X)$ and understand its meaning
- find the mean of any linear combination of random variables and the variance of any linear combination of independent random variables.

Ch3 - A model for friction (Integral link)

- A model for friction (p59)
- Modelling with friction (p60)

KEY POINTS

1 The total contact force between two surfaces may be expressed in terms of a frictional force and a normal reaction.

2 The frictional force, F, between two surfaces is given by:
$F < \mu R$ when there is no sliding except in limiting equilibrium
$F = \mu R$ in limiting equilibrium
$F = \mu R$ when sliding occurs
where R is the normal reaction of one surface on the other and μ is the coefficient of friction between the surfaces.

3 The frictional force always acts in the direction to oppose sliding.

4 The magnitude of the normal reaction is affected by any force which has a component perpendicular to the direction of sliding.

LEARNING OUTCOMES

When you have completed this chapter, you should

- understand that the total contact force between surfaces may be expressed in terms of a frictional force and a normal reaction
- be able to draw a force diagram to represent a situation involving friction
- understand that a frictional force may be modelled by $F \le \mu R$
- know that a frictional force acts in the direction to oppose sliding
- be able to model friction using $F = \mu R$ when sliding occurs
- know how to apply Newton's laws of motion to situations involving friction
- be able to derive and use the result that a body on a rough slope inclined at angle α to the horizontal is on the point of slipping if $\mu = \tan \alpha$.

Ch4 Moments of forces (Integral link)

- Introduction to moments (p71)
- The moment of a force which acts at an angle (p83)
- Sliding and toppling (p93) (Integral link)

KEY POINTS

1 The moment of a force F about a point O is given by the product Fd where d is the perpendicular distance from O to the line of action of the force.

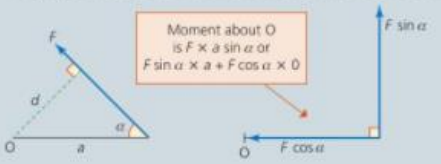

Figure 4.74

2 The S.I. unit for moment is the newton metre (N m)

3 Anticlockwise moments are usually called positive, clockwise negative.

4 If a body is in equilibrium the resultant force is zero and the sum of the moments of the forces acting on it, about any point, is zero.

LEARNING OUTCOMES

When you have completed this chapter, you should

- be able to calculate the moment about a fixed point O of a force acting on a body as the product of the force and the perpendicular distance of O from the line of action of the force or by first resolving the force into components and then finding the product of that component which does not go through O and its distance from O.
- be able to find the resultant of a set of parallel forces
- know how different types of lever work
- know the meaning of the word couple
- know that an object is in equilibrium if the resultant of all the applied forces acting on it is zero and the sum of their moments about any point is also zero
- be able to identify how equilibrium can be broken by sliding or toppling.

Ch3 - Discrete probability distributions (Integral link)

- The binomial distribution (p45)
- The Poisson distribution (p49)
- Link between binomial and Poisson distributions (p59)
- Other discrete distributions (p64)

KEY POINTS

Binomial distribution

1 The binomial distribution may be used to model situations in which these conditions hold
- You are conducting trials on random samples of a certain size, n.
- On each trial the outcomes can be classified as either success or failure.

For the binomial distribution to be a good model, these assumptions are required.
- The outcome of each trial is independent of the outcome of any other trial.
- The probability of success is the same on each trial.

2 For a binomial random variable X, where $X \sim B(n, p)$
- $P(X = r) = {}^nC_r q^{n-r} p^r$ for $r = 0, 1, 2, \ldots, n$.

3 For $X \sim B(n, p)$
- $E(X) = np$
- $Var(X) = npq$.

Poisson distribution

1 The Poisson distribution may be used in situations in which:
- the variable is the frequency of events occurring in fixed intervals of time or space.

2 For the Poisson distribution to be a good model:
- events occur randomly
- events occur independently
- events occur at a uniform average rate.

3 For a Poisson random variable X, where $X \sim Poisson(\lambda)$
- $P(X = r) = e^{-\lambda} \times \frac{\lambda^r}{r!}$ for $r = 0, 1, 2, \ldots$

4 For $Poisson(\lambda)$
- $E(X) = \lambda$
- $Var(X) = \lambda$.

5 The sum of two independent Poisson distributions
- If $X \sim Poisson(\lambda)$ and $Y \sim Poisson(\mu)$, then $X + Y \sim Poisson(\lambda + \mu)$.

Uniform distribution

1 The uniform distribution may be used to model situations in which:
- all outcomes are equally likely.

2 For a uniform random variable X
- $P(X = r) = \frac{1}{n}$ for $r = 1, 2, \ldots, n$.

3 For a uniform random variable X
- $E(X) = \frac{n+1}{2}$
- $Var(X) = \frac{n^2 - 1}{12}$.

Geometric distribution

1 The geometric distribution may be used in situations in which:
- there are two possible outcomes, often referred to as success and failure
- both outcomes have fixed probabilities, p and q and $p + q = 1$
- you are finding the number of trials which it takes for the first success to occur.

2 For the geometric distribution to be a good model:
- the probability of success is constant
- the probability of success in any trial is independent of the outcome of any other trial.

3 For a geometric random variable X, where $X \sim Geo(p)$
- $P(X = r) = (1-p)^{r-1} p = q^{r-1} p$ for $r = 1, 2, 3 \ldots$

4 For $Geo(p)$
- $E(X) = \frac{1}{p}$
- $Var(X) = \frac{1-p}{p^2}$.

LEARNING OUTCOMES

When you have completed this chapter you should be able to:

- recognise situations under which the binomial distribution is likely to be an appropriate model
- calculate probabilities using a binomial distribution
- know and be able to use the mean and variance of a binomial distribution
- recognise situations under which the Poisson distribution is likely to be an appropriate model
- calculate probabilities using a Poisson distribution
- know and be able to use the mean and variance of a Poisson distribution
- know that the sum of two or more independent Poisson distributions is also a Poisson distribution
- recognise situations in which both the Poisson distribution and the binomial distribution might be appropriate models
- recognise situations under which the discrete uniform distribution is likely to be an appropriate model
- calculate probabilities using a discrete uniform distribution
- calculate the mean and variance of any given discrete uniform distribution
- recognise situations under which the geometric distribution is likely to be an appropriate model
- calculate the probabilities using a geometric distribution, including cumulative probabilities
- know and be able to use the mean and variance of a geometric distribution.

Spring - Half Term 4

Teacher 1 - Mechanics

Ch5 Work, energy and power (Integral link)

- Energy and momentum (p101)
- Work and energy (p101)
- Gravitational potential energy (p109)
- Work and kinetic energy for two-dimensional motion (p103)
- Power (p108)

KEY POINTS

1 The work done by a constant force F is given by Fs where s is the distance moved in the direction of the force.
2 The kinetic energy (K.E.) of a body of mass m moving with speed v is given by $\frac{1}{2}mv^2$. Kinetic energy is the energy a body possesses on account of its motion.
3 The work–energy principle states that the total work done by all the forces acting on a body is equal to the increase in the kinetic energy of the body.
4 The gravitational potential energy of a body of mass m at a height h above a given reference level is given by mgh. It is the work done against the force of gravity in raising the body.
5 Mechanical energy (kinetic energy + gravitational potential energy) is conserved when no forces other than gravity do work.
6 Power is the rate of doing work, and is given by Fv.
7 Average power = total work done ÷ total time taken
8 The S.I. unit for energy is the joule and for power is the watt.

LEARNING OUTCOMES

When you have completed this chapter, you should be able to

- understand the language relating to work, energy and power
- calculate the work done by a force which moves either along its line of action or at an angle to it
- calculate kinetic energy and gravitational potential energy
- understand and use the principle of conservation of energy
- understand and use the work–energy principle
- understand and use the concept of power.

Ch6 Impulse and momentum (Integral link)

- Impulse (p127)
- Conservation of momentum (p131)
- Newton's law of impact (p138)

KEY POINTS

1 The impulse from a force **F** is given by **F**t where t is the time for which the force acts.
2 Impulse is conventionally denoted by **J**. It is a vector quantity.
3 The momentum of a body of mass m travelling with velocity **v** is given by m**v**. Momentum is a vector quantity.
4 The S.I. unit of impulse and momentum is the newton second (N s)
5 The impulse–momentum equation is

impulse = final momentum – initial momentum

6 The law of conservation of momentum states that when no external forces are acting on a system, the total momentum of the system is constant. Since momentum is a vector quantity this applies to the magnitude of the momentum in any direction.
7 Newton's law of impact:

Coefficient of restitution $e = \dfrac{\text{speed of separation}}{\text{speed of approach}}$

speed of separation = e × speed of approach

LEARNING OUTCOMES

When you have completed this chapter, you should

- know how to apply the principle of conservation of momentum to direct impacts
- understand Newton's law of impact and know the meaning of coefficient of restitution
- know and use the fact that $0 \le e \le 1$
- understand the implications of values of 0 and 1 for the coefficient of restitution
- understand that when the coefficient of restitution is less than 1, energy is not conserved during an impact
- be able to find the loss of kinetic energy during a direct impact
- know that for perfectly elastic collisions there is no energy loss
- know that for perfectly inelastic collisions the energy loss is the largest it can be.

Teacher 2 - Statistics

Ch 4 - Bivariate data (correlation coefficients) (Integral link)

- Describing variables (p76)
- Interpreting scatter diagrams (p77)
- Product moment correlation (p79)
- Rank correlation (p101)

KEY POINTS

1 A scatter diagram is a graph to illustrate bivariate data.
2 Notation for n pairs of observations (x, y)

$$S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x^2 - n\bar{x}^2$$

$$S_{yy} = \Sigma(y_i - \bar{y})^2 = \Sigma y^2 - n\bar{y}^2$$

$$S_{xy} = \Sigma(x_i - \bar{x})(y_i - \bar{y}) = \Sigma xy - n\bar{x}\bar{y}$$

3 Pearson's product moment correlation coefficient

$$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\Sigma(x_i - \bar{x})^2 \times \Sigma(y_i - \bar{y})^2}} = \frac{\Sigma x_i y_i - n(\bar{x})(\bar{y})}{\sqrt{(\Sigma x_i^2 - n\bar{x}^2)(\Sigma y_i^2 - n\bar{y}^2)}}$$

4 Spearman's coefficient of rank correlation

$$r_s = 1 - \frac{6\Sigma d_i^2}{n(n^2 - 1)}$$

5 Hypothesis testing based on Pearson's product moment correlation coefficient

H_0: $\rho = 0$

H_1: $\rho > 0$ or $\rho < 0$ (one-tailed test) or $\rho \ne 0$ (two-tailed test).

Test the sample value, r, against the critical value, which depends on the number of pairs in the bivariate sample, n, and the significance level.

6 Hypothesis testing based on Spearman's coefficient of rank correlation

H_0: no association

H_1: positive association or negative association (one-tailed test) or some association (two-tailed test).

Test the sample value, r_s, against the critical value, which depends on the number of pairs in the bivariate sample, n, and the significance level.

LEARNING OUTCOMES

When you have completed this chapter you should:

- understand what bivariate data are and know the conventions for choice of axis for variables in a scatter diagram
- be able to use and interpret a scatter diagram
- interpret a scatter diagram produced by software
- be able to calculate Pearson's product moment correlation coefficient from raw data or summary statistics
- know when it is appropriate to carry out a hypothesis test using Pearson's product moment correlation coefficient
- be able to carry out hypothesis tests using the Pearson's product moment correlation coefficient and tables of critical values or the p-value from software
- use the Pearson's product moment correlation coefficient as an effect size
- be able to calculate Spearman's rank correlation coefficient from raw data or summary statistics
- be able to carry out hypothesis tests using Spearman's rank correlation coefficient and tables of critical values or the output from software
- decide whether a test based on r or r_s may be more appropriate, or whether neither is appropriate.

Ch7 Centre of mass 1 (Integral link)

- The centre of mass (p148)
- Centre of mass of two- and three-dimensional bodies (p153)

KEY POINTS

1 The centre of mass of a body has the property that the moment, about any point, of the whole mass of the body taken at the centre of mass is equal to the sum of moments of the various particles comprising the body.

$$M\bar{\mathbf{r}} = \sum m_i \mathbf{r}_i \quad \text{where } M = \sum m_i$$

2 In one dimension

$$M\bar{x} = \sum m_i x_i$$

3 In two dimensions

$$M\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \sum m_i \begin{pmatrix}x_i\\ y_i\end{pmatrix}$$

4 In three dimensions

$$M\begin{pmatrix}\bar{x}\\ \bar{y}\\ \bar{z}\end{pmatrix} = \sum m_i \begin{pmatrix}x_i\\ y_i\\ z_i\end{pmatrix}$$

LEARNING OUTCOMES

When you have finished this chapter, you should

- be able to find the centre of mass of a system of particles of given position and mass
- be able to find the centre of mass of a simple shape
- know how to use symmetry when finding a centre of mass
- know the positions of the centres of mass of simple shapes
- be able to find the centre of mass of a composite body
- be able to use centre of mass in problems involving the equilibrium of a rigid body.

Ch8 Dimensional analysis (Integral link)

- Introduction to dimensional analysis (p167)
- The dimensions of further quantities (p168)
- Other systems of units (p169)
- Dimensional consistency (p171)
- Finding the form of the relationship (p172)
- The method of dimensions (p174)

KEY POINTS

1 Any quantity in mechanics may be expressed in terms of the three fundamental dimensions: mass, M; length, L; time, T.
2 The dimension of a quantity d is denoted by $[d]$.
3 The unit for any quantity is derived from the three fundamental dimensions.
4 Numbers are dimensionless.
5 All formulae and equations must be dimensionally consistent.
6 Using dimensional analysis you can sometimes find the form of a relationship as the product of powers of the quantities involved and a dimensionless constant.

LEARNING OUTCOMES

When you have completed this chapter, you should be able to

- find the dimensions of a quantity in terms of M, L and T
- use the dimensions of a quantity to determine its units
- change the units in which a quantity is given
- understand that some quantities are dimensionless
- use dimensional analysis to check a relationship for consistency
- use dimensional analysis to determine unknown indices in a proposed formula
- use a model based on dimensional analysis.

Practice questions: Set 1 (p181)

Ch 5 Bivariate data (regression lines) (Integral link)

- The least squares regression line (random on non-random) (p115)
- The least squares regression line (random on random) (p124)

KEY POINTS

Random on non-random regression lines

1 The equation of the y on x regression line $y = a + bx$ is given by

$$y - \bar{y} = b(x - \bar{x}), \text{ where } b = \frac{S_{xy}}{S_{xx}} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2} = \frac{\Sigma xy - n\bar{x}\bar{y}}{\Sigma x^2 - n\bar{x}^2}$$
$$\Rightarrow a = \bar{y} - b\bar{x}$$

2 For any data pair (x, y) the predicted value of y is

$$\hat{y} = a + bx \Rightarrow \text{the residual is } \varepsilon = y - \hat{y}$$

3 The sum of the residuals $\Sigma\varepsilon = 0$.
4 The least squares regression line minimises the sum of the squares of the residuals, $\Sigma\varepsilon^2$.
5 When using a regression line for prediction, a value within the data values (interpolation) is more likely to be predicted reliably than a value beyond the data values (extrapolation).

Random on random regression lines

6 If a value of y is to be predicted from a value of x, then the y on x regression line $y = a + bx$ needs to be used. The equation of this regression line is exactly the same as for the random on non-random regression line.
7 If a value of x is to be predicted from a value of y, then the x on y regression line $x = a + by$ needs to be used. The equation for this line is given by

$$x - \bar{x} = b(y - \bar{y}), \text{ where}$$
$$b = \frac{S_{xy}}{S_{yy}} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(y-\bar{y})^2} = \frac{\Sigma xy - n\bar{x}\bar{y}}{\Sigma y^2 - n\bar{y}^2}$$
$$\Rightarrow a = \bar{x} - b\bar{y}$$

8 Both of these lines pass through the point $(\bar{x}, \bar{y})$.
9 The goodness of fit of the regression line can be judged by the coefficient of determination. This is the square of the value of Pearson's product moment correlation coefficient.

LEARNING OUTCOMES

When you have completed this chapter you should be able to:

- obtain the equation of the least squares regression line for a random variable on a non-random variable, using raw data or summary statistics
- use the regression line as a model to estimate a value of the random variable and know when it is appropriate to do so
- know the meaning of the term residual and be able to calculate and interpret residuals
- obtain the equations of the two least squares regression lines, y on x and x on y, where both variables are random, using raw data or summary statistics
- use either regression line to estimate the expected value of one variable for a given value of the other and know when it is appropriate to do so
- check how well the model fits the data
- know the relationship between the two regression lines and when to use one rather than the other.

	Ch 6 Chi-squared tests [(Integral link)] - The chi-squared test for a contingency table (p133) - Goodness of fit tests (p147)

KEY POINTS

1 Contingency tables

To test whether the variables in an $m \times n$ contingency table are independent the steps are as follows.

(i) The null hypothesis is that the variables are independent, the alternative is that they are not.
(ii) Calculate the marginal (row and column) totals for the table.
(iii) Calculate the *expected frequency* in each cell.
(iv) The X^2 statistic is $\Sigma \frac{(f_o - f_e)^2}{f_e}$ where f_o is the observed frequency and f_e is the expected frequency in each cell.
(v) The *degrees of freedom*, υ, for the test is $(m-1)(n-1)$ for an $m \times n$ table.
(vi) Read the critical value from the χ^2 tables (alternatively, use suitable software) for the appropriate degrees of freedom and significance level. If X^2 is less than the significance level, the null hypothesis is accepted; otherwise it is rejected.
(vii) If two variables are not independent, you say that there is an *association* between them.

2 Goodness of fit tests

To test whether a distribution models a situation, the steps are as follows.

(i) Select your model, binomial, Poisson, etc.
(ii) Set up null and alternative hypotheses and choose the significance level.
(iii) Collect data. Record the observed frequency for each outcome.
(iv) Calculate the expected frequencies arising from the model.
(v) Check that the expected frequencies are all at least 5. If not, combine classes.
(vi) Calculate the test statistic.
The X^2 statistic is $\Sigma \frac{(f_o - f_e)^2}{f_e}$.
(vii) Find the degrees of freedom, υ, for the test using the formula.
υ = number of classes − number of estimated parameters − 1
where the number of classes is counted *after* any necessary combining has been done.
(viii) Read the critical value from the χ^2 tables for the appropriate degrees of freedom and significance level. If X^2 is less than the significance level, the null hypothesis is accepted; otherwise it is rejected.
(ix) Draw conclusions from the test – state what the test tells you about the model.

LEARNING OUTCOMES

When you have completed this chapter you should be able to:

➤ interpret bivariate categorical data in a contingency table
➤ apply the χ^2 test to a contingency table
➤ carry out a χ^2 test for goodness of fit of a uniform, binomial, geometric or Poisson model
➤ interpret the results of a χ^2 test using tables of critical values or the output from software.

Practice questions: Set 1 (p165)

Summer - Half Term 5

Teacher 1 - Mechanics	Teacher 2 - Statistics
Revision for AS exam	**Revision for AS Exam**