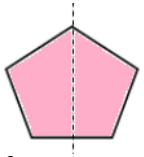
| Knowledge Organiser | Subject | Year | Term | KO n.o. | Title |
|---|---|---|---|---|---|
| | Mathematics | 9 | 3 | 9B | Ch 9 Transformations |

## Lines of symmetry



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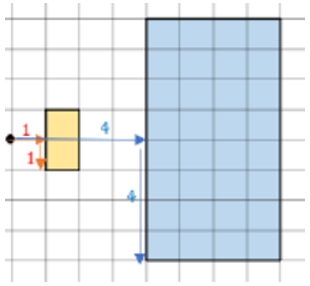


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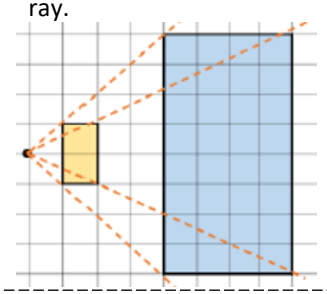

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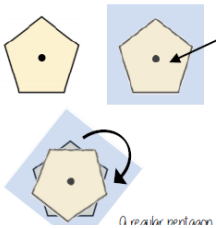
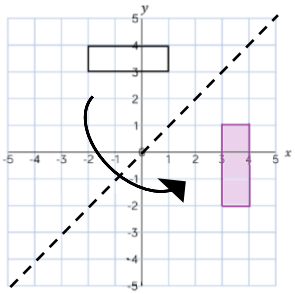


## Reflect Diagonally



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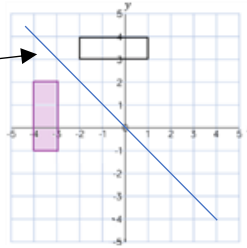

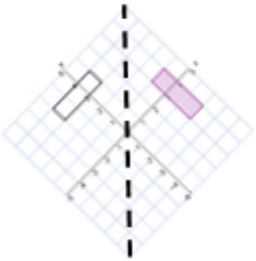

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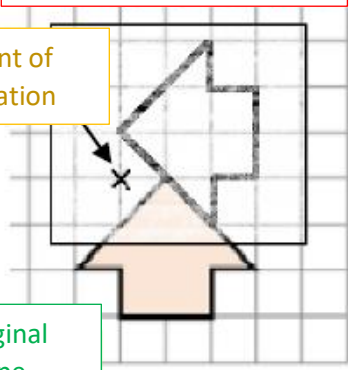
## Rotation from a point (outside the shape)

Image 90° anti clockwise

Point of rotation

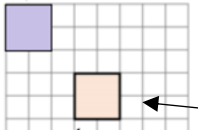


Original shape

1. Trace the original shape
2. Keep the point in the same place and turn the tracing paper
3. Draw the new shape

## Translation:

Every **vertex** has been translated by the same amount in the same direction



Translation 3 units left and 3 units up

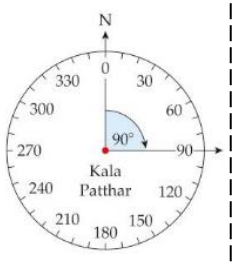
Original shape

Vector notation
How far left (negative) or right (positive)
How far Down (negative)
or up (positive)

$$\begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

## Bearings

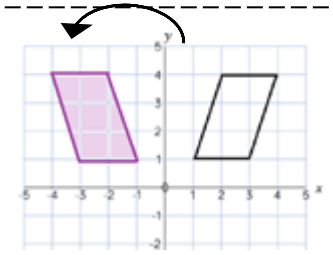
When you give a three-figure bearing
- Measure from North
- Measure clockwise
- Use 3 digits




The angle of one place from the other called the **bearing**.

## Reflect horizontally or vertically

All points need to be the same distance away from the line of reflection. Reflection in the line y axis – this is also a reflection in the line x = 0



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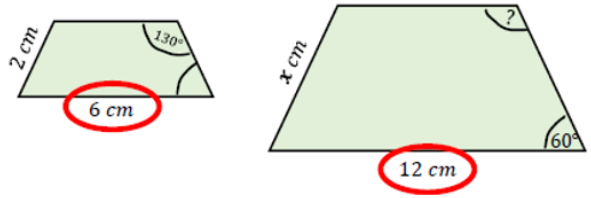
## Calculations in similar shapes

The two trapeziums are similar. Find the missing side and angle



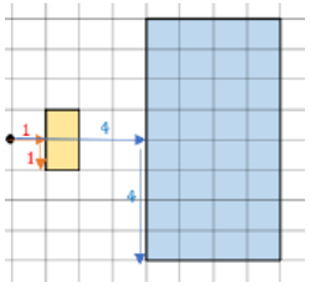

Corresponding sides identify the scale factor so 12/6 = 2. SF is 2

The missing side is 2 x 2 = 4 so x = 4cm

Enlargement does not change angle size so corresponding angles are equal so ? is 130°

## Enlarge a shape from a point

Method 1: Scale the distance between the point of enlargement and each corresponding vertex.

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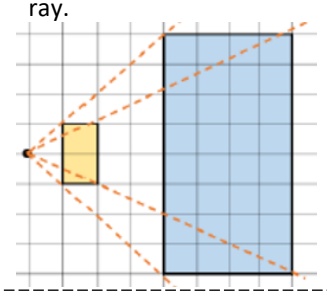

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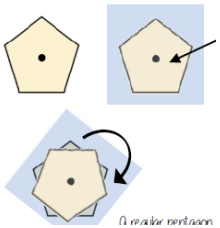

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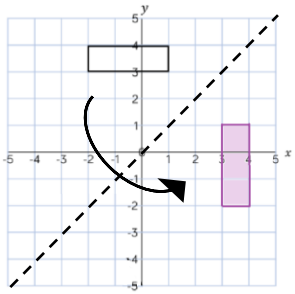

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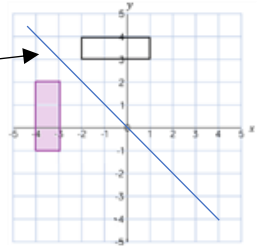

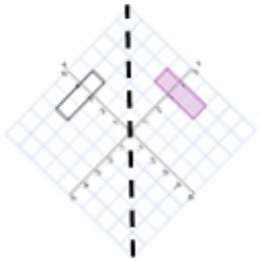

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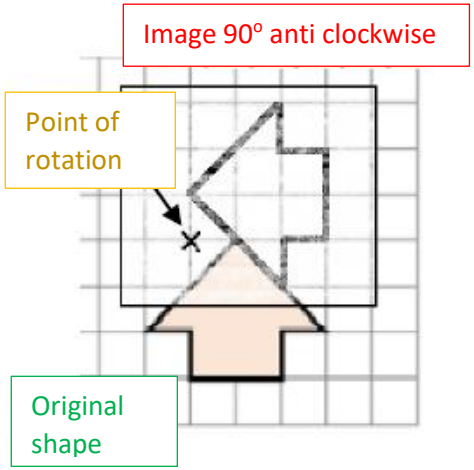


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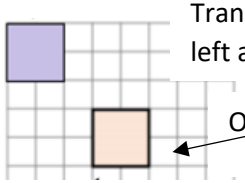

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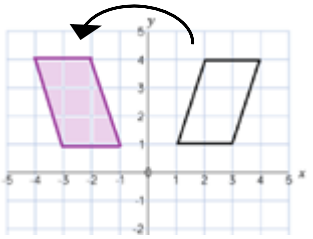

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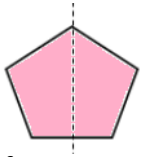

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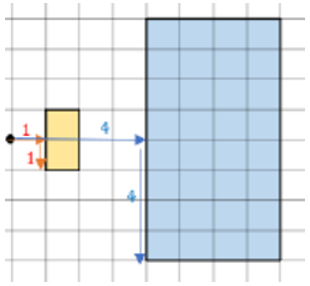


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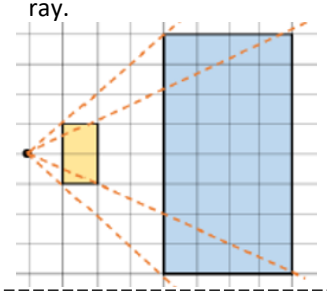

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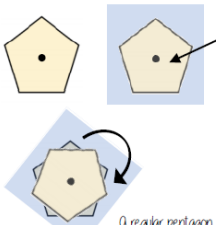

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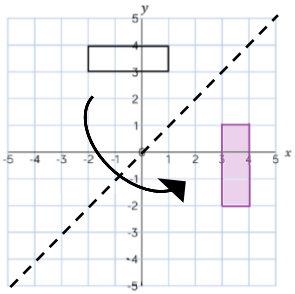

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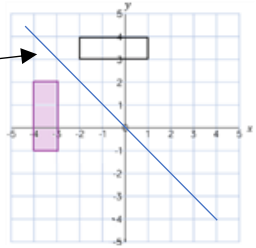

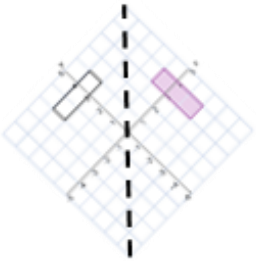

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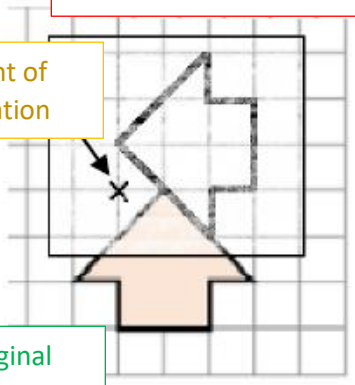

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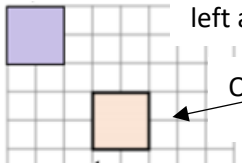
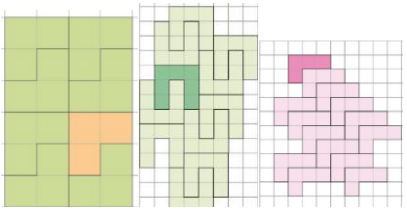


Translation 3 units left and 3 units up

Original shape

## Tessellation

A tiling pattern with no gaps or overlaps. These shapes are all congruent in these diagrams



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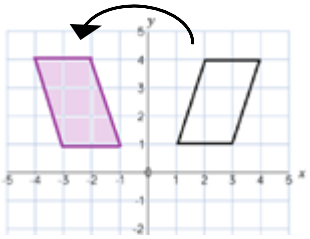

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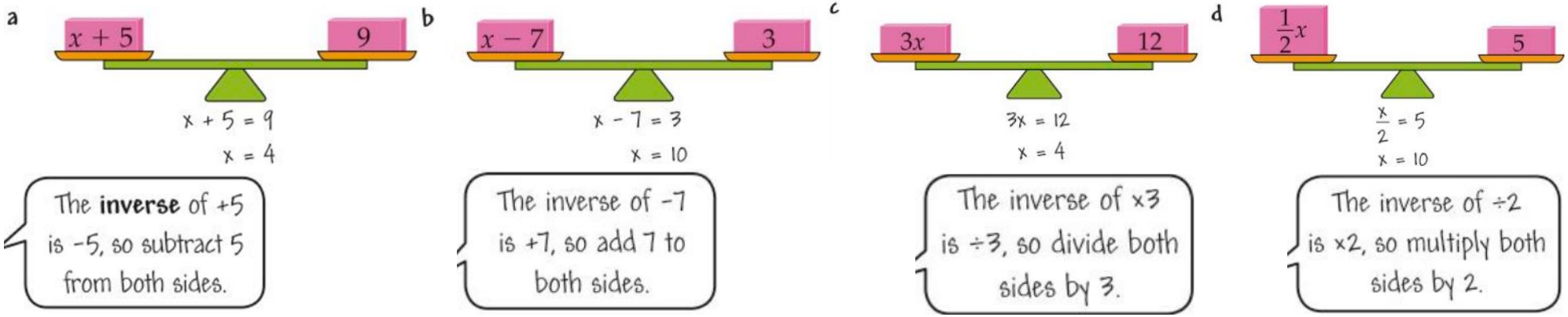
|---|---|---|---|---|---|
| | Mathematics | 9 | 2 | 10A | Ch 10 Equations |

## Solving 1 step equations

Solving equations is where you find the value of the unknown.

Solve these equations.

**a** $x + 5 = 9$ **b** $x - 7 = 3$ **c** $3x = 12$ **d** $\frac{1}{2}x = 5$




## Problem solving

a) Write an equation using the amounts of money in this story. Ivy has £x in her wallet. She spends £5. She is left with £15.

b)

c) Solve your equation to find out how much money Ivy had at the start.

a) $x - 5 = 15$

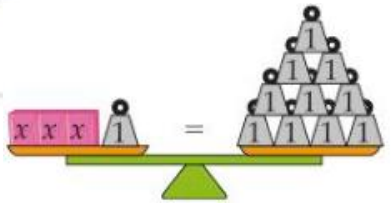
b) $x = 20$ so Ivey had £20 at the start.

## Solving 2 step Equations

Solving 2 step equations is where you find the value of the unknown using two stages to do so.

**a** Write an equation for these scales.

**b** Solve the equation by balancing.




**a** The equation is $3x + 1 = 10$.

**b**

$3x + 1 = 10$

$3x + 1 - 1 = 10 - 1$ Subtract 1 from both sides.

$3x = 9$

$3x \div 3 = 9 \div 3$ Divide both sides by 3.

$x = 3$

| Knowledge Organiser | Subject | Year | Term | KO n.o. | Title |
|---|---|---|---|---|---|
| | Mathematics | 9 | 2 | 10B | Ch 10 Equations |

## Solving 1 or 2 step equations

Solving equations is where you find the value of the unknown.

Solve the equations.

**a** $x - 9 = 23$

a The inverse of − is +,
So add 9 to both sides

$x - 9 + 9 = 23 + 9$

$x = 32$

**b** $4x = 52$

b The inverse of × is ÷,
So divide both sides by 4

$\frac{4x}{4} = \frac{52}{4}$

$x = 13$

Solve the equation $3x + 8 = 2$

Subtract 8 from both sides.

Divide both sides by 3.

$3x + 8 = 2$

$3x = -6$

$x = -2$

Solve the equation $\frac{x}{2} - 5 = 3$

Add 5 to both sides.

× both sides by 2.

$\frac{x}{2} - 5 = 3$

$\frac{x}{2} = 8$

$x = 16$

## Unknowns on both sides

Solve these equations.

**a** $\frac{1}{3}(7x - 1) = 2x + 3$

a $\frac{1}{3}(7x - 1) = 2x + 3$

Multiply both sides by 3.

Subtract 6x from both sides.

Add 1 to both sides.

$7x - 1 = 6x + 9$

$x - 1 = 9$

$x = 10$

**b** $\frac{4x - 3}{5} = x - 2$

b $\frac{4x - 3}{5} = x - 2$

Multiply both sides by 5.

Subtract 4x from both sides

Add 10 to both sides.

Rearrange.

$4x - 3 = 5x - 10$

$-3 = x - 10$

$7 = x$

$x = 7$

## Equations with brackets

Solve $3(5x + 2) - 2(4x - 3) = 26$.

Expand the brackets

Collect like terms

Subtract 12 from both sides

Divide both sides by 7

$3(5x + 2) - 2(4x - 3) = 26$

$15x + 6 - 8x + 6 = 26$

$7x + 12 = 26$

$7x = 14$

$x = 2$

| Knowledge Organiser | Subject | Year | Term | KO n.o. | Title |
|---|---|---|---|---|---|
| | Mathematics | 9 | 2 | 10C | Ch 10 Equations |

## Solving Linear equations

Solving equations is where you find the value of the unknown.

Solve $10 - 5x = 12$

$10 - 5x = 12$

$10 = 12 + 5x$ Add $5x$ to both sides.

$-2 = 5x$

$x = -\frac{2}{5}$ Divide both sides by 5.

Solve $\frac{4}{2-3x} = \frac{5}{6-2x}$

$\frac{4}{2-3x} = \frac{5}{6-2x}$

$4(6-2x) = 5(2-3x)$

$24 - 8x = 10 - 15x$

$24 + 7x = 10$

$7x = -14$

$x = -2$

## Solving Simultaneous equations graphically

Draw the graphs of $2x - y = 5$ and $x + 2y = 5$. Find their point of intersection. What does this point represent?

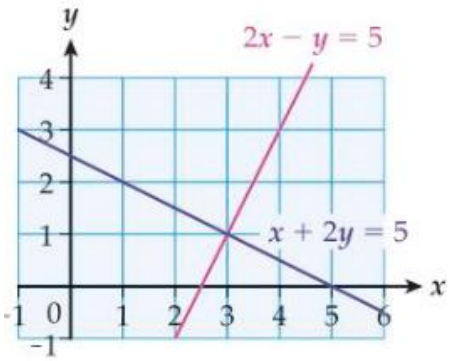
$2x - y = 5$

$\Rightarrow y = 2x - 5$

| x | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| y | -5 | -3 | -1 | 1 |

$x + 2y = 5$

| x | 0 | 5 |
|---|---|---|
| y | 2.5 | 0 |




The graphs intersect at (3,1) so x = 3 and y = 1 is the solution.

## Solving Inequalities

Solve these inequalities and represent the solutions on a number line.

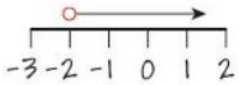
**i** $5x + 5 > 2x + 1$ **ii** $-5y \geq -20$

**a i** $5x + 5 > 2x - 1$ $-2x$

$3x + 5 > -1$ $-5$

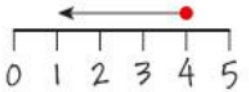
$3x > -6$ $\div 3$

$x > -2$



**ii** $-5y \geq -20$

$y \leq 4$



## Simultaneous Equations

To Eliminate a variable, you may have to multiply one or both Simultaneous equations to make the coefficient the same.

Solve the following pairs of simultaneous equations.

$3x - 4y = 22$ (1)

$x + 4y = 18$ (2)

Eliminate the $y$-terms because they have matching **coefficients**.

(1) + (2) $4x + 0 = 40$ They have opposite signs so add.

$\Rightarrow x = 10$

$10 + 4y = 18$ Substitute $x = 10$ into (2).

$\Rightarrow 4y = 8$ and $y = 2$

$3 \times 10 - 4 \times 2 = 30 - 8$ Check using (1).

$= 22$ ✓

Solve the following pairs of simultaneous equations

**a** $x + 3y = 16$ (1)

$2x - y = -3$ (2)

Eliminate the $x$-terms.

$2 \times (1)$ $2x + 6y = 32$ (3)

All terms have been doubled.

$2x - y = -3$ (2)

(3) − (2) $7y = 35$

$\Rightarrow y = 5$

and $x = 1$

**b** $5x - 3y = 13$ (1)

$3x + 2y = 4$ (2)

Eliminate the $y$-terms.

$2 \times (1)$ $10x - 6y = 26$ (3)

$3 \times (2)$ $9x + 6y = 12$ (4)

(3) + (4) $19x = 38$

$\Rightarrow x = 2$

and $y = -1$

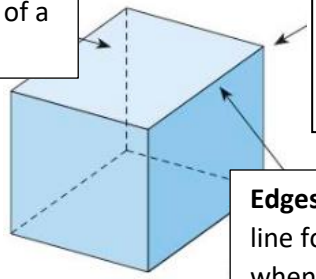
| Knowledge Organiser | Subject | Year | Term | KO n.o. | Title |
|---|---|---|---|---|---|
| | Mathematics | 9 | 2 | 14A | Ch 14 3D shapes |

## Describing 3D Shapes

You can describe a 3D shape by its faces, edges and vertices.

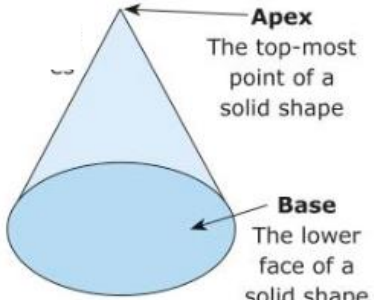
**Face**: A flat surface of a solid

**Vertex:** The point three or more edges meet



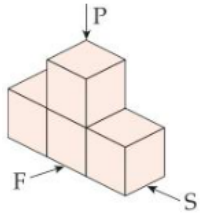
**Edges:** The line formed when two faces meet

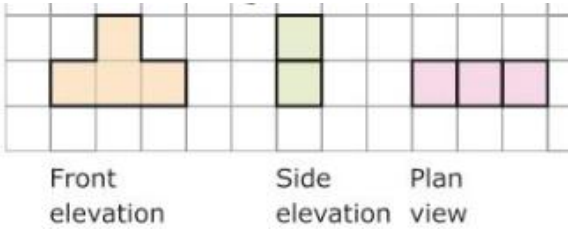
For some it is apex and base.




## Plans and Elevations

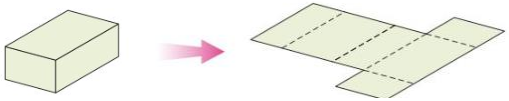
A **Front elevation** is the view from the front.
A **Side elevation** is the view form the side.
A **Plan** is the view from above.



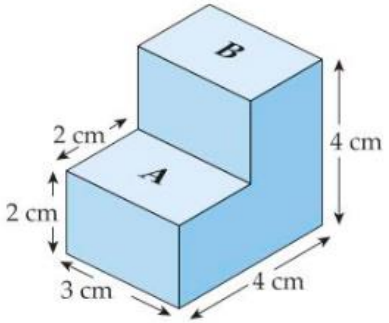

## Nets

When a 3D shape is unfolded the flat shape is called a net.

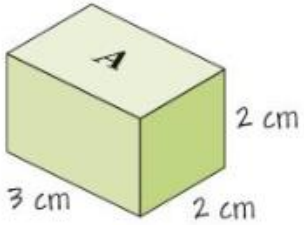


## Volume of shapes made from Cuboids

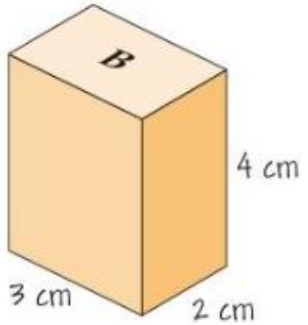
Find the volume of this shape.




Split the shape into simpler cuboids.




Volume of A = 3 x 2 x 2
= 12 cm³



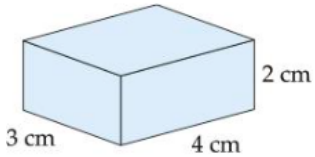

Volume of B = 4 x 3 x 2
= 24 cm³

The total volume is
12 cm³ + 24 cm³ = 36 cm³.

## Surface Area of Cuboids

Calculate the surface area of the cuboid.

Area of top is 3 x 4 = 12 $cm^2$
Area of side = 4 x 2 = 8 $cm^2$
Area of front = 3 x 2 = 6 $cm^2$
Total = 2 x (12 + 8 + 6) = 52$cm^2$

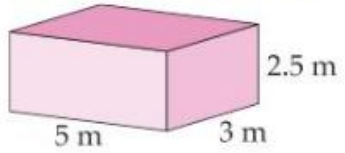



## Volume

Volume is a measure of how space a 3D shape occupies.
It can be calculated by volume = length x width x height

Volume = 5 x 3 x 2.5 = 37.5 $cm^3$

Calculate the volume of this cuboid.



| Knowledge Organiser | Subject | Year | Term | KO n.o. | Title |
|---|---|---|---|---|---|
| | Mathematics | 9 | 2 | 14B | Ch 14 3D shapes |

## Describing 3D Shapes

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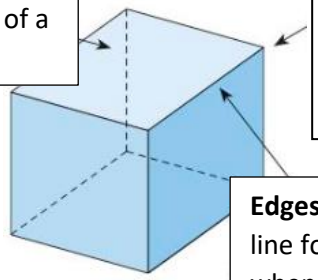
**Vertex:** The point three or more edges meet



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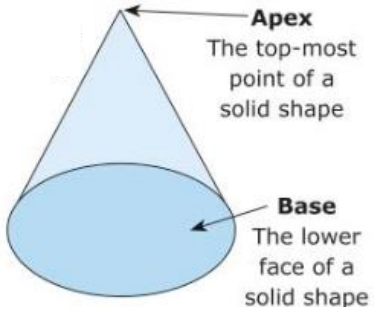
For some it is apex and base.

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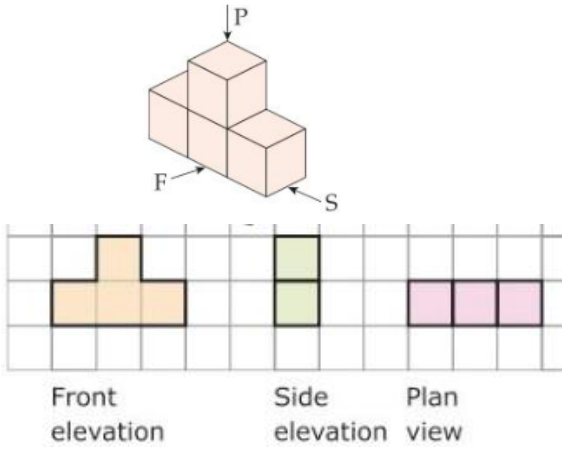
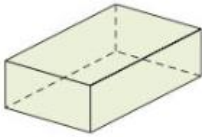



## Symmetry of 3D shapes

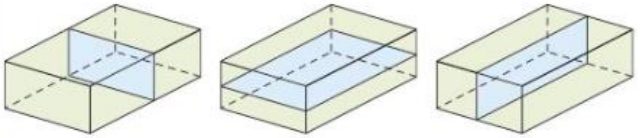
A line of symmetry divides a 2D shape into 2 identical halves.

A **plane** of symmetry divides a 3D shape into 2 identical halves. Each is the mirror of each other.

Draw diagrams to show all the planes of symmetry of this cuboid.

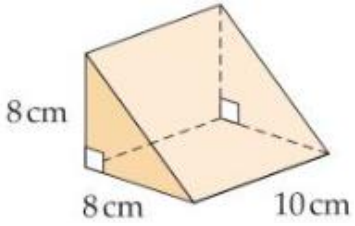


The cuboid has three planes of symmetry.



## Volume of a prism

Volume of a prism = area of cross section x length



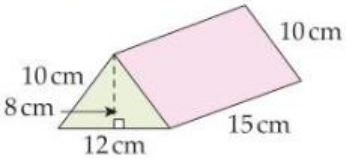

Area of triangle $= \frac{1}{2} \times 8 \times 8$
$= 32\ cm^2$

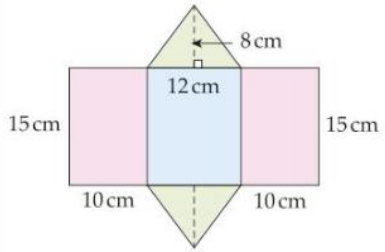
Volume of the prism = area of cross-section × length
$= 32 \times 10$
$= 320\ cm^3$

## Surface Area of Prisms

The surface area of a prism is the area of the individual faces added together.

Calculate the surface area of this triangular prism.




| | | |
|---|---|---|
| Area of one pink rectangle | $= 15 \times 10$ | $= 150\ cm^2$ |
| Area of another pink rectangle | $= 15 \times 10$ | $= 150\ cm^2$ |
| Area of the blue rectangle | $= 15 \times 12$ | $= 180\ cm^2$ |
| Area of one green triangle | $= \frac{1}{2} \times 12 \times 8$ | $= 48\ cm^2$ |
| Area of another green triangle | $= \frac{1}{2} \times 12 \times 8$ | $= 48\ cm^2$ |
| **Total surface area** | | $= 576\ cm^2$ |

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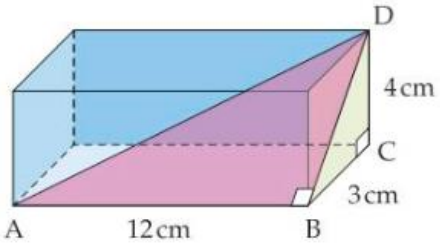
|---|---|---|---|---|---|
| | Mathematics | 9 | 2 | 14C | Ch 14 3D shapes and Trigonometry |

## 3D Geometry

Calculate the length of AD




In triangle BCD

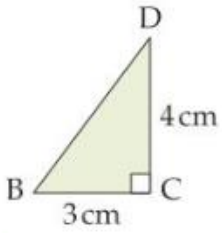
$BD^2 = 3^2 + 4^2$

$= 9 + 16$

$BD^2 = 25$

$(BD = \sqrt{25} = 5\,cm)$

$AD^2 = AB^2 + BC^2 + CD^2$



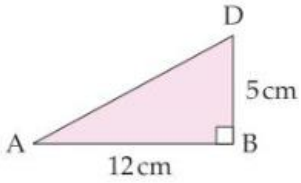

In triangle ABD

$AD^2 = 5^2 + 12^2$

$= 25 + 144$

$= 169$

$AD = \sqrt{169} = 13\,cm$

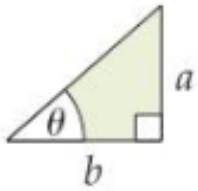



## Trigonometry

The **hypotenuse** is the longest side of a right-angled triangle.

The **opposite** sides faces the angle marked Ɵ.

The **adjacent** side is beside the angle marked Ɵ.




$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

## Symmetry of 3D shapes

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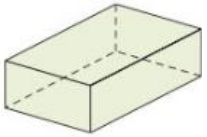
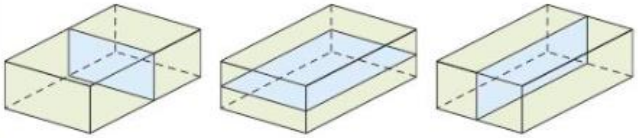
Draw diagrams to show all the planes of symmetry of this cuboid.

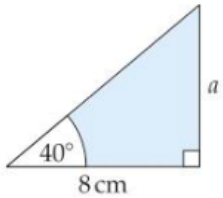


The cuboid has three planes of symmetry.



## Trigonometric Questions Lengths

Calculate the lengths *a*




Tan uses opposite and adjacent.

$$\tan 40° = \frac{opp}{adj} = \frac{a}{8}$$

$a = 8 \times \tan 40° = 6.7\,cm$ (1 dp)

## Bearings

A **three-figure bearing** is the angle

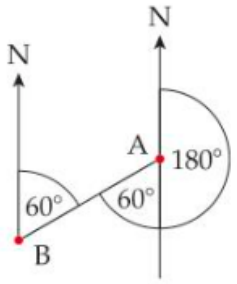
- measured from north (N)
- in a clockwise direction
- and written with three digits

The bearing of

a to b is 060°

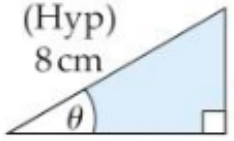
The reverse bearing is

180° + 60° = 240°




## Trigonometric Questions - Angles

Sin uses opposite and hypotenuse.




$$\sin\theta = \frac{opp}{hyp} = \frac{4}{8} = 0.5$$

$\theta = \sin^{-1} 0.5 = 30°$

| Knowledge Organiser | Subject | Year | Term | KO n.o. | Title |
|---|---|---|---|---|---|
| | Mathematics | 9 | 2 | 15A | Ch 15 Ratio and Proportion |

## Ratio

You can compare the size of two quantities by writing them as a **ratio**. They can be simplified by dividing both sides by the same number.

You can make an equivalent ratio by multiplying both sides of the **ratio** by the same number

Simplify the ratio 35 : 63

The common factor of 35 and 63 is 7.

35 : 63 → ÷7 → 5 : 9

35 : 63 simplifies to 5 : 9

## Ratio and Proportion

**Proportiona**l relationships always keep the same **ratio**. This is **Direct Proportion.**

Ali gets paid £6 per hour he works. How much does he get paid for 5 hours?

time : wage

1 : 6 → ×5 → 5 : 30

Ali earns £30 when he works for 5 hours.

## Financial Maths

You can calculate which offer is best to buy by calculating price per item.

Brand A
You end up getting
1kg for £1.20

Brand B
You get 1 kg for
90p + 45 = 135p = £1.35


Buy 1 get 1 free | Buy 1 get 1 half price

So the offer on Brand A is better value for money

## Dividing Into Ratios

To share an amount into **ratios** you need to add the parts together to find the total number of parts.
Divide the amount by the total parts.
This value is then multiplied by the number in each grouping.

Divide 20 sweets in the ratio 2 : 3.

2 : 3 → 5 parts

20 ÷ 5 = 4

2 : 3 → ×4 → 8 : 12

## Percentage and Proportion

You can compare proportions by converting them to **percentages**. Or find the fractions of the amounts to compare.

Jemima's maths score is $\frac{90}{100}$ which is 90%.

Her English score is $\frac{85}{100}$ which is 85%.

So Jemima did better in her maths test than her English test.

## Ratio and Proportion

A **proportion** compares the size of the part to the whole.
A **ratio** compares the size of 2 or more parts.
You simplify ratios where possible.

This is the national flag of Nigeria.

The ratio green : white = 2 : 1

The proportion of green = $\frac{2}{3}$

The proportion of white = $\frac{1}{3}$

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| | Mathematics | 9 | 2 | 15B | Ch 15 Ratio and Proportion |

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35 : 63 simplifies to 5 : 9

## Ratio and Proportion

**Proportiona**l relationships always keep the same **ratio**. This is **Direct Proportion.** Often you will use the **Unitary Method**

Ali gets paid £6 per hour he works. How much does he get paid for 5 hours?

5 litres of petrol cost £5.75.
What is the cost of 18 litres of petrol?

5 litres £5.75 → ÷5 / ÷5 →
1 litre £1.15 → ×18 / ×18 →
18 litres £20.70

## Financial Maths

You can calculate which offer is **best to buy** by calculating price per item.


Buy 1 get 1 half price

Buy 2 get the 3rd one free

Brand A: 2 bags £1.40 + £0.70 = £2.10 for 1 kg

Brand B 3Bags: 2 x £1.50 = £3.00 for 1.5kg

£3.00 ÷ 1.5 = £2.00 per kg

So Brand B is better value

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Divide the amount by the total parts.
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Divide 20 sweets in the ratio 2 : 3.

2 : 3 → 5 parts

20 ÷ 5 = 4

2 : 3 → ×4 / ×4 → 8 : 12

## Comparing Proportions

You can compare proportions by converting them to **percentages**. Or find the fractions of the amounts to compare.

Here are two different chocolate bars.


Bar A Bar B

Which bar contains the higher proportion of fat?

First write the proportion of fat in each bar as a fraction.

| Chocolate bar A | Chocolate bar B |
|---|---|
| $\frac{80}{250}$ | $\frac{36}{120}$ |

Then convert each fraction into a percentage.

| | |
|---|---|
| = 80 ÷ 250 | = 36 ÷ 120 |
| = 0.32 | = 0.3 |
| = 32% | = 30% |

Chocolate bar A contains the higher proportion of fat.

## Uses of Ratio

You can compare **ratios** by changing them into the form **1:n**.

Class 9A has the ratio of boys to girls of 4:5. Class 9B has the ratio of boys to girls of 9:11 Which has the higher proportion of girls?

Class 9A: boys : girls 4 : 5 → ÷4 / ÷4 → 1 : 1.25

Class 9B: boys : girls 9 : 11 → ÷9 / ÷9 → 1 : 1.22 (2 dp)

So 9A has the higher proportion of girls

| Knowledge Organiser | Subject | Year | Term | KO n.o. | Title |
|---|---|---|---|---|---|
| | Mathematics | 9 | 2 | 15C | Ch 15 Ratio and Proportion |

## Ratio

You can compare the size of two quantities by writing them as a **ratio**. They can be simplified by dividing both sides by the same number.

Simplify the ratio. zinc : copper

3 : 4.5

× 2 ↓ ↓ × 2

6 : 9

÷ 3 ↓ ↓ ÷ 3

2 : 3

## Financial Maths

You can calculate which offer is **best to buy** by calculating price per item.


Size X: £0.99 x 4 =£3.96 for 1 litre

Size Y: £1.80 x 2 = £3.60 for 1 litre

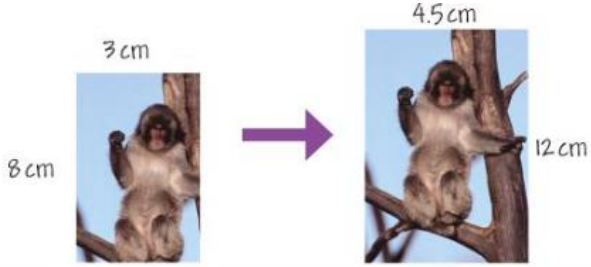
Size Z: £2.50 x (4/3) = £3.33 for 1 litre

So Size Z is cheapest per litre so best value.

## Proportional Reasoning

When two quantities are in **direct proportion** you can use the **ratio** to change from one quantity to another using a **multiplier**.

A picture is enlarged by a scale factor of 1.5. What is the ratio of the lengths form original to the enlargement?




original : enlarged

= 3 : 4.5 = 8 : 12

= 2 : 3

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Ali gets paid £6 per hour he works. How much does he get paid for 5 hours?

5 litres of petrol cost £5.75. What is the cost of 18 litres of petrol?

5 litres £5.75

÷ 5 ↓ ↓ ÷ 5

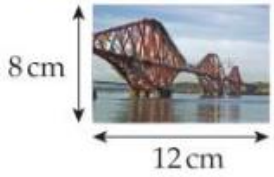
1 litre £1.15

× 18 ↓ ↓ × 18

18 litres £20.70

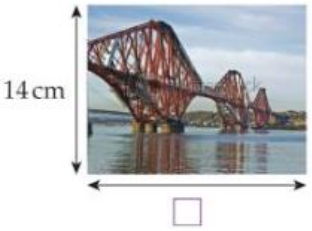
## Proportion and scales

Calculate the length of the larger photograph.




Length ÷ width = scale factor

12 ÷ 8 = 1.5




So for the larger photo

Length = 1.5 x width

14 x 1.5 = 21 cm

## Fractions and Proportions

You can calculate a fractional increase or decrease in a single calculation using a **multiplier.**

Decrease £235 by $\frac{1}{5}$

The price has decreased by $\frac{1}{5}$

New price

$= (1 - \frac{1}{5})$ of the old price

$= \frac{4}{5}$ of the old price

$= \frac{4}{5} \times 235$

b Increase £235 by $\frac{1}{5}$

b The price has increased by $\frac{1}{5}$

New price

$= (1 + \frac{1}{5})$ of the old price

$= \frac{6}{5}$ of the old price

$= \frac{6}{5} \times 235$

| Knowledge Organiser | Subject | Year | Term | KO n.o. | Title |
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| | Mathematics | 9 | 2 | 16A | Ch 16 Probability |

## Probability Introduction

Some things are certain, some impossible and some are uncertain. **Probability** describes how likely a future event is. Every **probability** is a number between **0 and 1.**
An **event** is certain if it **always** happen.
An **event** is **likely** if it happens quite often.
An **event** has an **evens** chance if you would expect it to happen half the time.
An **event** is **unlikely** if it does not happen very often.
An **event** is **impossible** if it cannot happen.
An event is **random** if its outcomes are **uncertain**

## Mutually Exclusive

Two events are **mutually exclusive** if it is impossible for them to happen at the same time.

If you know the probability of an event occurring, p, then the probability of it not happening is 1 – p.
Outcomes are **exhaustive** if between them they include all possible outcomes of an event.

## Listing Outcomes

Using a table can make it easier to list the outcomes in a systematic way.
A table can be used to list all the outcomes of 2 events in a systematic way is called a **sample space diagram**

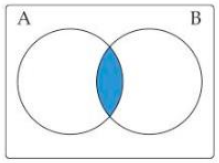
| Skirts \ Shirts | Blue | Red |
|---|---|---|
| Blue | Blue Blue | Red Blue |
| Orange | Blue Orange | Red Orange |
| White | Blue White | Red White |

## Ven Diagrams

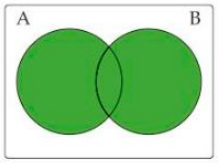
### Venn Diagram Language

The **universal set**, Ω, is the set containing all the elements.
**A ∩ B** means the elements in A **and** B
**A U B** means the elements in A **or** B
**A' U B'** means the elements not in A or not in B

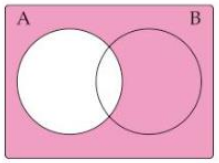
A ∩ B means 'the **intersection** of A and B'.




A ∪ B means 'the **union** of A and B'




A' means 'the **complement** of A'.




## Experimental Probability

We can often use an **experiment** to **estimate** probability. An experiment is a set of **trials**. The more trials the **more reliable** the estimation is. If you repeat an experiment, you will probably get different outcomes.

Experimental proability = $\frac{\text{Number of successful trials}}{\text{Total number of trials}}$

Sam watched 20 cars approaching a junction and recorded the direction they turned. 8 turned left and 12 right. Estimate the probability the next car will turn left.

Probability of turning left = 8/20 = 4/ 10 = 0.4

## Theoretical Probability

Probability of an event = $\frac{\text{Number of equally likely outcomes}}{\text{Total number of outcomes}}$

If you roll an ordinary dice, what is the probability of getting
**a** a score of 5 **b** an even number?

a) There is 1 5 on the dice with 6 outcomes so probability is 1/6
b) There are 3 even numbers out of the 6 outcomes so probability of even is 3/6 = 1/2

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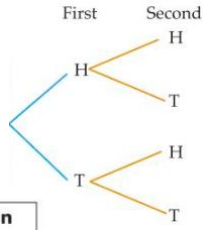
## Listing outcomes

A **trial** is a statistical experiment, like throwing a dice.
An **outcome** is a possible result of a trial, like throwing a 5.
An **event** is a collection of outcomes, like throwing an odd number ( 1, 3 or 5)
A tree diagram shows possible outcomes.




A sample space diagram shows outcomes.

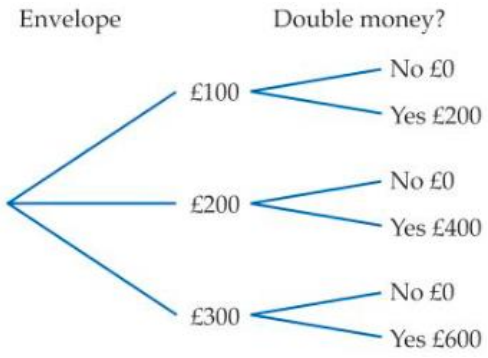
| | | Second coin | |
|---|---|---|---|
| | | **Heads** | **Tails** |
| **First coin** | **Heads** | (H, H) | (H, T) |
| | **Tails** | (T, H) | (T, T) |

## Tree Diagrams

Archie wins a competition and gets to pick one of three identical envelopes which contain £100, £200 or £300. He then must flip a coin. If it lands on heads, he doubles his prize money. If it lands on tails, he wins nothing.

**a** Show all the possible outcomes in a tree diagram.
**b** **i** What is the probability that he wins a least £300?
**ii** What is the probability that he wins nothing?

**a**




**b** **i** $P(\text{wins} \geq £300) = \frac{2}{6} = \frac{1}{3}$

**ii** $P(\text{wins nothing}) = \frac{3}{6} = \frac{1}{2}$

## Mutually Exclusive

Two outcomes are **mutually exclusive** if they cannot happen at the same time.

If a set of **mutually exclusive** events covers all possible outcomes, then their sum of probabilities is 1.

## Experimental Probability

We can often use an **experiment** to **estimate** probability. An experiment is a set of **trials**. The more trials the **more reliable** the estimation is. If you repeat an experiment, you will probably get different outcomes.

Experimental proability = $\frac{\text{Number of successful trials}}{\text{Total number of trials}}$

Sam watched 20 cars approaching a junction and recorded the direction they turned. 8 turned left and 12 right. Estimate the probability the next car will turn left.

Probability of turning left = 8/20 = 4/ 10 = 0.4

## Venn Diagram Language

The **universal set**, Ω, is the set containing all the elements.
**A ∩ B** means the elements in A **and** B
**A U B** means the elements in A **or** B
A' **U** B' means the elements not in A or not in B

If A is a subset of B if:

- Every element in A is also contained in B
- Set A does not contain every element in B.

We write this as A ⊂ B

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## Independent Events

If an outcome does not affect what happens in another then the events are **independent**.

If A and B are independent events **then P(A and B both happen) = P(A) x P(B)**

A road safety officer estimates that $\frac{1}{4}$ of bicycles used by teenagers do not have properly inflated tyres and $\frac{1}{6}$ of them don't have adequate lights.

**a** If these are independent, what is the probability that a bicycle belonging to a teenager, chosen at random, has both faults?

**b** Do you think independence is reasonable?

**a** P(both faults) $= \frac{1}{4} \times \frac{1}{6} = \frac{1}{24}$. While the tyres and lights are not directly connected, they may both be dependent on how careful the owner is, so independence is not likely to be completely correct.

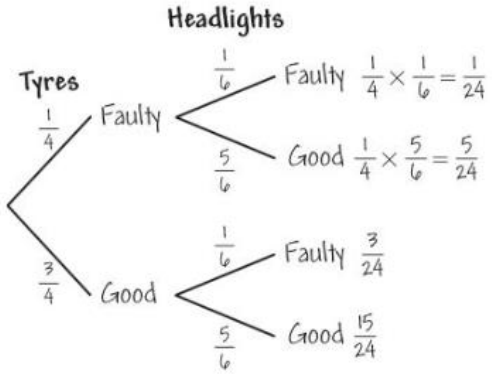
## Probability of Combined Events

If independent:

**P(A and B both happen) = P(A) x P(B)**

If mutually exclusive:

P(A or B) = P(A)+P(B)

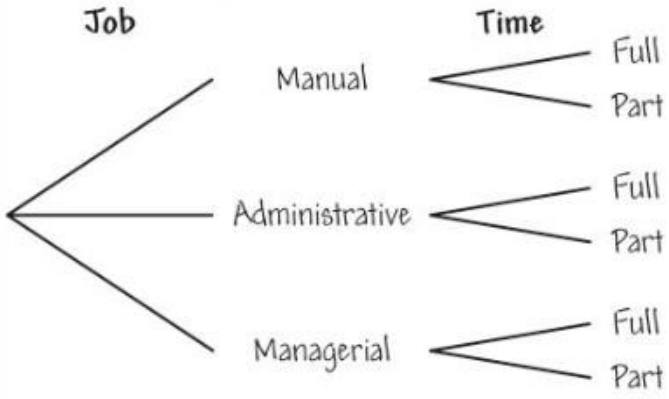



$$P(\text{fail one test}) = \frac{5}{24} + \frac{3}{24} = \frac{8}{24} = \frac{1}{3}$$

## Tree Diagrams

A **tree diagram** can be used to show the possible outcomes for combined events. It is particulary useful when outcomes are not equally likely.

There is no limit to the number of possible outcomes at any stage.




## Venn Diagram Language

The **universal set**, Ω, is the set containing all the elements.

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A' **U** B' means the elements not in A or not in B

Two events are mutually exclusive if $A \cap B = \varnothing$.

A is a proper subset of B, $A \subset B$, if

- every element in A is also contained in set B
- set A does not contain every element in B.

If $A \subset B$ then $P(A) < P(B)$.

Two events are independent if $P(A \cap B) = P(A) \times P(B)$.